Axiomatizing Congestion Control

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The overwhelmingly large design space of congestion control protocols, along with the increasingly diverse range of application environments, makes evaluating such protocols a daunting task. Simulation and experiments are very helpful in evaluating the performance of designs in *specific* contexts, but give limited insight into the more general properties of these schemes and provide no information about the *inherent* limits of congestion control designs (such as, which properties are simultaneously achievable and which are mutually exclusive). In contrast, traditional theoretical approaches are typically focused on the design of protocols that achieve to specific, predetermined objectives (e.g., network utility maximization), or the analysis of specific protocols (e.g., from control-theoretic perspectives), as opposed to the inherent tensions/derivations between desired properties.

To complement today's prevalent experimental and theoretical approaches, we put forth a novel principled framework for reasoning about congestion control protocols, which is inspired by the axiomatic approach from social choice theory and game theory. We consider several natural requirements ("axioms") from congestion control protocols – e.g., efficient resource-utilization, loss-avoidance, fairness, stability, and TCP-friendliness – and investigate which combinations of these can be achieved within a single design. Thus, our framework allows us to investigate the fundamental tradeoffs between desiderata, and to identify where existing and new congestion control architectures fit within the space of possible outcomes.

CCS Concepts: • **Networks** → Network protocols;

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1 Introduction

Recent years have witnessed a revival of both industrial and academic interest in improving congestion control designs. The quest for better congestion control is complicated by the extreme diversity and range of (i) the design space (as exemplified by the stark conceptual and operational differences between recent proposals [8, 14, 18, 19, 49, 50]), (ii) the desired properties (ranging from high performance to fairness to TCP-friendliness), (iii) the envisioned operational setting (inter- and intra-datacenter, wireless, the commercial Internet, satellite), and (iv) the application loads and requirements (small vs. large traffic demands, latency- *vs.* bandwidth-sensitive).

Most congestion control research uses simulation and experiments under a limited range of network conditions. This is extremely important for understanding the detailed performance of

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particular schemes in specific settings, but provides limited insight into the more general properties of these schemes and no information about the *inherent* limits (such as, which properties are simultaneously achievable and which are mutually exclusive). In contrast, traditional theoretical approaches are typically focused on the design of protocols that achieve specific, predetermined objectives (e.g., network utility maximization [26, 46]), or the analysis of specific protocols (e.g., from control-theoretic perspectives [1, 37]), as opposed to exploring the inherent tensions/derivations between desired properties.

We advocate an axiomatic approach to congestion control, which is complementary to the experimental and theoretical work currently being pursued. Our approach, modeled on similar efforts in social choice theory and game theory [7], identifies a set of requirements ("axioms") and then identifies (i) which of its subsets of requirements can coexist (i.e., there are designs that achieve all of them) and which subsets cannot be met simultaneously (i.e., no design can *simultaneously* achieve all of them), and (ii) whether some requirements immediately follow from satisfying other requirements. Thus, the axiomatic approach can shed light on the *inherent* tradeoffs involved in congestion control protocol design, and can be leveraged to classify existing and proposed solutions according to the properties they satisfy.

The axiomatic approach has been applied to many computer science environments, e.g. reputation systems [47], recommendation systems [6], link prediction [17], and networking environments [17, 28, 42]. To the best of our knowledge, ours is the first application of this approach to congestion control protocols (though [42] touches on the subject briefly).

We introduce a simple network model where we can evaluate congestion control designs and formulate several natural axioms (or requirements) for congestion control protocols, including efficient link-utilization, loss-avoidance, fairness, stability, and TCP-friendliness. Congestion control protocols can be regarded as points in a multidimensional space reflecting the extent to which they satisfy these requirements, and we show how classical families of congestion control protocols (e.g., additive-increase-multiplicative-decrease [16, 53], multiplicative-increase-multiplicative-decrease, and more) can be mapped to points in this space.

We leverage our axiomatic framework to derive basic results on the feasibility of *simultaneously* achieving different requirements within a single design. Our results formalize and shed light on various empirical/experimental observations about tensions between different desiderata, including (1) the tension between attaining high performance and being friendly to legacy TCP connections [27, 36], (2) the tension between achieving high bandwidth and maintaining low latency under dynamic environments [11, 32, 50], and (3) the tension between being robust to non-congestion loss and not incurring high loss upon convergence [19]. From a protocol design perspective, desirable congestion control protocols are those that reside on the Pareto frontier in the multidimensional space induced by our requirements, and this Pareto frontier is characterized by our theoretical results.

To be sure, our axiomatic approach has its limitations, as it revolves around investigating these properties in a simplified model. However, we feel that the results, in terms of which axioms can coexist and which cannot, provide insights that apply far beyond the simple model (even if the detailed theoretical results do not). Thus, we contend that the axiomatic approach is a useful addition to the evaluatory arsenal that researchers should apply to congestion control.

Organization. In the following section, we introduce the simple network model we use to evaluate congestion control designs and discuss its limitations. Then, in Section 3, we formulate several natural axioms (or requirements) for congestion control protocols, including efficient link-utilization, loss-avoidance, fairness, stability, and TCP-friendliness. In Section 4, we derive our basic results on

the feasibility of achieving these requirements and then, in Section 5, we show the mapping from congestion control protocols to points in a multidimensional space.

2 Modeling Protocol Dynamics

We consider a simple model of dynamics of congestion control protocols (which builds upon [9, 16, 36]). We show below that studying our simple (and tractable) model gives rise to interesting insights about the fundamental tradeoffs involved in congestion control protocol design. We also discuss the limitations of our model.

Senders dynamically interacting on a link. *n senders* $1, \ldots n$ send traffic on a link of bandwidth $B > 0$ (measured in units of MSS/s), propagation delay Θ, and buffer size τ (measured in units of MSS).[1] In our model, B, Θ, and τ are all unknown to the senders, precluding the possibility of building into protocols a priori assumptions, such as "the link is only shared by senders running protocol P", "the minimum latency is at least $2ms$", etc. We let $C = B \times 2\Theta$, i.e., C represents the minimum possible bandwidth-delay product for the link, or its "capacity". Senders experience *synchronized* feedback. Time in our model is regarded as an infinite sequence of discrete time steps $t = 0, 1, 2, \ldots$, each of RTT duration, in which end-host (sender) decisions occur. At the beginning of each time step t, sender i can adjust its congestion window by selecting a value in the range $\{0, 1, \ldots, M\}$, in units of MSS. We assume that $1 << M$. Let $x_i^{(t)}$ denote the size of the sender i's congestion window at time t, let $\bar{x}^{(t)} = (x_1^{(t)}, \ldots, x_n^{(t)})$, and let $X^{(t)} = \sum_i x_i^{(t)}$.

The duration of time step t, $RTT(t)$, is a function of the link's propagation delay and the buffer's queuing delay. This is captured (as in [27, 31, 53]) as follows:

$$RTT(\bar{x}^{(t)}, C, \tau) = \begin{cases} \max(2\Theta, \frac{X^{(t)}-C}{B} + 2\Theta), \\ \text{if } X^{(t)} < C + \tau \\ \Delta, \quad \text{otherwise} \end{cases} \tag{1}$$

where Δ represents a timeout-triggered cap on the RTT in the event of packet loss.

When traffic sent on the link at time t exceeds $C + \tau$, excess traffic is dropped according to the FIFO (droptail) queuing policy. The *loss rate* experienced by each sender at time step t as a function of the congestion window sizes of all senders and the link capacity and buffer size, is captured as:

$$L(\bar{x}^{(t)}, C, \tau) = \begin{cases} (1 - \frac{C+\tau}{X^{(t)}}) & \text{if } X^{(t)} > C + \tau \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

To simplify exposition, we will denote the loss rate at time t simply by $L^{(t)}$.

Congestion control protocols and the induced dynamics. Each sender i's selection of congestion-window sizes is dictated by the *congestion control protocol* P_i employed by that sender. A congestion control protocol (deterministically) maps the history of congestion-window sizes of that sender, and of the RTTs and loss rates experienced by that sender, to the sender's next selection of congestion window size. A protocol is *loss-based* if its choice of window-sizes is invariant to the RTT values. Any choice of initial congestion windows $\bar{x}^{(0)} = (x_1^{(0)}, \ldots, x_n^{(0)})$ and congestion control protocol P_i for each sender i induces a *dynamic* of congestion control as follows: senders start sending as in $\bar{x}^{(0)}$ and, at each time step, every sender i repeatedly reacts to its environment as prescribed by P_i.

Protocols belonging to the families of Additive-Increase-Multiplicative-Decrease (**AIMD(a,b)** [21, 52]) and Multiplicative-Increase-Multiplicative-Decrease (**MIMD(a,b)** [4, 5]) are easy to model in our framework: AIMD(a,b) increases the window size $x_i^{(t)}$ additively by a (MSS) if the loss $L^{(t)}$ at time t is 0, whereas MIMD(a,b) increases the window size multiplicatively by a factor of a. Both

[1] MSS (maximum segment size) is the maximum number of bytes in a single TCP segment.

protocols multiplicatively decrease the window size by a factor of b if $L^{(t)} > 0$. We formalize two more prominent families of protocols within our model: Binomial [9], and TCP Cubic [24].

A binomial protocol **BIN(a, b, k, l)** for $a > 0,\ 0 < b \leq 1,\ k \geq -1,\ l \in [0, 1]$ is defined as follows:

$$x_i^{(t+1)} = \begin{cases} x_i^{(t)} + \frac{a}{(x_i^{(t)})^k} & \text{if } L^{(t)} = 0 \\ x_i^{(t)} - b(x_i^{(t)})^l & \text{if } L^{(t)} > 0 \end{cases}$$

TCP Cubic, **CUBIC(c, b)**, can be captured as follows:

$$x_i^{(t+1)} = \begin{cases} x_i^{max} + c(T - (\frac{x_i^{max}(1-b)}{c})^{\frac{1}{3}})^3 & \text{if } L^{(t)} = 0 \\ x_i^{max} b & \text{if } L^{(t)} > 0 \end{cases}$$

where x_i^{max} is the window size of sender i when the last packet loss was experienced, T is the number of time steps elapsed from the last packet loss, $b \in (0, 1)$ is the rate-decrease factor, and $c > 0$ is a scaling factor.

Limitations of model. Clearly, our model does not capture many of the intricacies of congestion control. However, as will be shown in the sequel, our model encompasses many scenarios of interest, and trends revealed by our axiomatic investigation are also reflected in our experimental findings.

Aspects whose incorporation into our model is left for future research include: (1) Extending our model and results to network-wide dynamics (as opposed to competition over a single bottleneck link)[2]. (2) Modeling the effects of queuing at routers by incorporating ideas from queueing theory and control theory. (3) Examining other queueing policies (beyond the prevalent droptail), such as RED [10], WFQ [44], and SPQ [29]. (4) Extending our model to non-synchronized feedback [43, 51]. (5) Modeling slow-starts and timeouts. (6) Transitioning from a fluid-level model to a packet-level model to better capture, e.g., low-rate transmissions. (7) Extending our protocol formulation to incorporate randomized protocols. (8) Extending our model to pacing-based (as opposed to window-based) protocols (e.g., PCC [18, 19] and BBR [14]).

3 Axiomatic Approach

We consider eight natural requirements ("axioms") from congestion control protocols. Intuitively, these requirements capture the desiderata of well-utilizing spare network resources, avoiding excessive loss, achieving fairness amongst competing flows, converging to a stable rate-configuration, attaining high-performance in the presence of non-congestion loss [14], being friendly towards legacy TCP connections, and not suffering excessive latency. However, what precisely is the definition of "well-utilizing a link"? Is it utilizing 90% of the link from some point onwards? 95%? What does it mean to be friendly to TCP? Is it not to exceed by over 1.5x the throughput of a TCP connection sharing the same link? 3x?

Different choices of values to plug into our requirements (resulting in different formulations of axioms) can have significant implications, to the extent of opposite answers to questions regarding whether two (or more) requirements can be satisfied simultaneously. To allow for a nuanced discussion of the interplay between axioms, instead of reasoning about axioms with hardwired constants, we *parameterize* the requirements (turning them into "metrics").

A congestion control protocol can be regarded as a point in the 8-dimensional space induced by the following metrics, according to its "score" in each metric.

[2]Importantly, reasoning about network-wide dynamics of congestion control is highly challenging even for *fixed-rate* senders [23].

Metric I: efficiency. We say that a congestion-control protocol P is *α-efficient* if when all senders employ P, for any initial configuration of senders' window sizes, there is some time step T such that from T onwards $X^{(t)} \geq \alpha C$.

Metric II: fast-utilization. Our next metric is intended to exclude protocols that take an unreasonable amount of time to utilize spare bandwidth (e.g., that increase the window size by a single MSS every 1,000 RTTs). Intuitively, the following definition of *α-fast-utilization* captures the property that, for any "long enough" time period, the protocol consumes link capacity at least as fast as a protocol that increases the congestion window by α MSS in each RTT. A congestion-control protocol P is *α-fast-utilizing* if there exists $T > 0$ such that if a P-sender i's window size is $x_i^{(t_1)}$ at time step t_1 and by time step $t_1 + \Delta_t$, for any $\Delta_t \geq T$, does not experience loss, nor increased RTT (if not loss-based), then $\Sigma_{t=t_1}^{t_1+\Delta_t}(x_i^{(t)} - x_i^{(t_1)}) \geq \frac{\alpha\Delta_t^2}{2}$.

Metric III: loss-avoidance. We say that a congestion-control protocol P is *α-loss-avoiding* if, when all senders employ P, for any initial configuration of senders' window sizes, there is some time step T such that from T onwards the loss rate $L^{(t)}$ is bounded by α (e.g., $\alpha = 0.01$ translates to not exceeding loss rate of 1%). We refer to protocols that are 0-loss-avoiding, i.e., protocols that, from some point onwards, do not incur loss, as "0-loss".

Metric IV: fairness. We say that a congestion-control protocol P is *α-fair* if when all senders use P and for any configuration of senders' window sizes, from some time $T > 0$ onwards, the average window size of each sender i is at least an α-fraction that of any other sender j.

Metric V: convergence. We say that a congestion-control protocol P is *α-convergent*, for $\alpha \in [0, 1]$, if there is a configuration of window sizes $(x_1^*, \ldots, x_n^*) \in [0, M]^n$ and time step T such that for any $t > T$ and sender $i \in N$, $\alpha x_i^* \leq x_i^{(t)} \leq (2 - \alpha)x_i^*$ (e.g., $\alpha = 0.9$ means that from some point onwards the window sizes are within 10% from a fixed point).

Metric VI: robustness to non-congestion loss. A natural demand from congestion control protocols is to be *robust* to loss that does not result from congestion [14, 18]. Clearly, formulating this requirement in all possible contexts is challenging. We focus on a simple, yet enlightening, scenario (used in [18] to motivate PCC): Suppose that a single sender i sends on a link of infinite capacity (so as to remove from consideration congestion-based loss). We say that a protocol P is α-robust if, when the sender experiences *random* packet loss rate of at most $\alpha \in [0, 1]$ at any time $t > 0$ time, then, for any choice of initial senders' window sizes, and for any value $\beta > 0$, there is some $T > 0$ such that for every $\bar{t} > T$, $x_i^{(\bar{t})} \geq \beta$ (i.e., non-congestion loss of rate at most α does not prevent utilization of spare capacity).

Metric VII: TCP-friendliness. We say that a protocol P is *α-friendly* to another protocol Q if, for any combination of sender-protocols such that some senders use P and others use Q, for every initial configuration of senders' window sizes, and for every P-sender i and Q-sender j, from some point in time $T > 0$ on wards, j's average window size is at least an α-fraction of i's average window size. Observe that friendliness, as defined above, is closely related to fairness (Metric IV), but fairness is with respect to many instantiations of the *same* protocol, whereas friendliness captures interactions between *different* protocols. We say that a protocol P is *α-TCP-friendly* if P is α-friendly towards AIMD(1,0.5) (i.e., TCP Reno).

Metric VIII: latency-avoidance. We say that protocol P is α-latency-avoiding if for sufficiently large link capacity C and buffer size τ, and regardless of sender's initial window sizes, when all senders on the link employ P, there is some time step T such that from T onwards $RTT(t) < (1+\alpha)2\Theta$. The term 2Θ captures the minimum possible RTT (twice the link's propagation delay).

Why these metrics? As the above long list of metrics indicates, congestion control protocol design is a remarkably complex task. Proposed protocols are expected to meet many requirements, including optimizing multiple facets of locally-experienced performance (bandwidth utilization, loss rate, self-induced latency), reaching desirable global outcomes (fast convergence, fairness), not being overly aggressive to legacy TCP, and more. One of our key contributions is putting forth a theoretical framework that formalizes these desiderata and enables both the investigation of their interdependence, and the "scoring" of existing/proposed protocols. Our metrics formalize classical requirements from congestion control protocols [20], which are often considered when evaluating these protocols (see, e.g.,[8, 18, 19, 50]).

While our metrics capture intuitive interpretations of conventional requirements from congestion control, clearly other formalizations of these metrics could be considered. Investigating how, within our axiomatic framework, relaxing/modifying our metrics influences the possibility/impossibility borderline for congestion control is important for informing protocol design. In social choice theory and game theory, revisiting the axiomatic model of classical results (e.g., Arrow's Theorem [7] and the Gibbard-Satterthwaite Theorem [22]) has yielded valuable insights. Our metrics and results thus constitute the first steps in such a discussion.

4 Axiomatic Derivations

We present below theoretical results highlighting the intricate connections between our metrics. Our results establish that some properties of a congestion control scheme are immediate consequences of other properties or, conversely, cannot be satisfied in parallel to satisfying other properties. The latter form of results exposes fundamental tensions between metrics, implying that attaining a higher "score" in one metric inevitably comes at the expense lowering the score in another. We leverage this insight in Section 5 to reason about the Pareto frontier for protocol design in the 8-dimensional metric space induced by our metrics.

4.1 Relating Axioms

We begin with the following simple observation:

Claim 1. *Any loss-based protocol that is* 0*-loss is not* α*-fast-utilizing for any* $\alpha > 0$.

To see why the above is not obvious, consider a protocol P that slowly increases its rate until encountering loss for the first time and then slightly decreases the rate so as to not exceed the link's capacity. While both 0-loss (from some point in time no loss occurs) and almost fully-utilizing the link, this protocol is not α-fast-utilizing for any $\alpha > 0$. The reason is that, for loss-based protocols, α-fast-utilization implies after sufficiently long time without packet loss, the protocol must increase its rate until encountering loss yet again. We now prove this formally.

Proof. (of Claim 1) Consider the scenario that a single sender is utilizing the link and employing protocol P. Suppose that the sender is 0-loss-avoiding then after, sufficient time, its window size must be upper bounded by some value $H \leq C + \tau$ (otherwise a loss event occurs infinitely many times). However, recall that any loss-based, α-fast-utilizing protocol P (for $\alpha > 0$) is associated with a time value $T > 0$ such that for any $t_1 > 0$, if no loss event occurs in the time interval $[t_1, t_1 + T]$, then $\sum_{t=t_1}^{t_1+T}(x^{(t)} - x^{(t_1)}) \geq \frac{\alpha T^2}{2}$. This implies that the P-sender must increase its window size by at least $\frac{\alpha T^2}{2}$ within the time period $[t_1, t_1 + T]$. However, this implies that for any period of $\bar{T}$ time steps in which no loss event occurs, such that $\frac{\alpha T^2}{2}\text{MSS} > (C + \tau)$ the sender must experience loss. This contradicts the definition of the upper bound H. □

We present below other, more subtle, connections between our metrics. All of our bounds apply across all possible network parameters (link capacity, buffer size) and number of senders, with the

exception of Theorem 3 (where the reliance on C and τ is explicit). We start with the following result, which relates convergence, fast-utilization, and efficiency.

Theorem 1. *Any protocol that is α-convergent and β-fast-utilizing, for some $\beta > 0$, is at least $\frac{\alpha}{2-\alpha}$-efficient.*

Proof. (of Theorem 1) Let protocol P be α-convergent and β-fast-utilizing, for some $\beta > 0$. Suppose, for point of contradiction, that protocol P is not $\frac{\alpha}{2-\alpha}$-efficient. This implies that for any $T > 0$ there is $t > T$ such that

$$\sum_{i\in[n]} x_i^{(t)} < \left(\frac{\alpha}{2-\alpha}\right) C \tag{3}$$

Since P is α-convergent, there exists a configuration of window sizes $(x_1^*, \ldots, x_n^*) \in [0, M]^n$ such that after sufficient time $\tilde{T}$, for each P sender i and for any $t > \tilde{T}$,

$$\alpha x_i^* \leq x_i^{(t)} \leq (2-\alpha)x_i^* \tag{4}$$

As P is β-fast-utilizing, the sum of senders' rates must exceed the link capacity C, otherwise P can experience no loss and no latency increase indefinitely without increasing its rate, violating the definition of β-fast-utilization. This, combined with (4), implies that for some large enough t

$$C < \sum_i x_i^{(t)} \leq \sum_i (2-\alpha)x_i^* \tag{5}$$

Equation (4) also implies that $x_i^* \leq \frac{x_i^{(t)}}{\alpha}$.

Putting everything together yields that for infinitely many values of t

$$C < (2-\alpha)\sum_i x_i^* \leq (2-\alpha)\sum_i \frac{x_i^{(t)}}{\alpha} =$$
$$= \frac{2-\alpha}{\alpha}\sum_i x_i^{(t)} < \frac{2-\alpha}{\alpha} \times \frac{\alpha}{2-\alpha} C = C$$

—a contradiction. The theorem follows. □

4.2 Two Upper Bounds on TCP-Friendliness

We next present results showing that protocols that satisfy certain desiderata are upper bounded in terms of their levels of TCP-friendliness.

Theorem 2. *Any loss-based protocol that is α-fast-utilizing and β-efficient is at most $\frac{3(1-\beta)}{\alpha(1+\beta)}$-TCP-friendly*

Proof sketch for Theorem 2: To prove the theorem, we first establish that any loss-based protocol that is α-fast-utilizing and β-efficient is no more TCP friendly than $AIMD(\alpha, \beta)$. Intuitively, this is because $AIMD(\alpha, \beta)$ increases its rate at the minimal pace required to attain α-fast-utilization, and decreases its rate at the maximum pace required to not violate β-efficiency (thus freeing up bandwidth for contending TCP flows). Our formalization of this intuition leverages arguments similar to those in [13, 36] to reason about the duration of the time interval between two consecutive loss events when the protocol under consideration and TCP Reno share the same link. The longer the intervals the less TCP Reno's aggressive backoff mechanism is triggered and so the less TCP Reno's throughput is harmed. We observe that $AIMD(\alpha, \beta)$ maximizes the length across all α-fast-utilizing and β-efficient protocols and is thus the friendliest to TCP among them. We then derive

the upper bound on the TCP-friendliness of α-fast-utilizing and β-efficient protocols simply by upper bounding the friendliness of $AIMD(\alpha, \beta)$ towards TCP Reno (i.e., $AIMD(1, \frac{1}{2})$). We point out that the upper bound on TCP-friendliness of Theorem 2 is, in fact, tight, as $AIMD(\alpha, \beta)$) is precisely $\frac{3(1-\beta)}{\alpha(1+\beta)}$-TCP-friendly (see also the analysis in [13]).

Theorem 3. *Any loss-based protocol that is α-fast-utilizing, β-efficient, and ϵ-robust, for $\epsilon > 0$, is at most $\frac{3(1-\beta)}{(4(\frac{C+\tau}{1-\epsilon})-\alpha)(1+\beta)}$-TCP friendly.*

Proof sketch for Theorem 3: Similarly to the proof of Theorem 2, the proof of Theorem 3 also relies on identifying a class of congestion control protocols, such that upper bounds on TCP-friendliness for these protocols extend to all other protocols. Again, this involves reasoning about the time intervals between consecutive loss events.

Specifically, we introduce a family of protocols, called Robust-AIMD, which can be regarded as a hybrid of traditional AIMD and PCC [18]. Under Robust-AIMD, time is divided into short (roughly 1 RTT) "monitor intervals". In each monitor interval, the sender sends at a certain rate and uses selective ACKs from the receiver to learn the resulting loss rate. Robust-AIMD uses an AIMD-like rule for adjusting transmission rate: the sender has a congestion window (similarly to TCP and unlike PCC) that is additively increased by a predetermined constant a (MSS) if the experienced loss rate is lower than a fixed constant $\epsilon > 0$, and multiplicatively decreased by a predetermined constant b if the loss rate exceeds ϵ. Thus, Robust-AIMD is modeled as follows. Robust-AIMD(a,b,ϵ):

$$x_i^{(t+1)} = \begin{cases} x_i^{(t)} + a & \text{if } L^{(t)} < \epsilon \\ x_i^{(t)} b & \text{if } L^{(t)} \geq \epsilon \end{cases}$$

We show that any loss-based protocol that is α-fast-utilizing, β-efficient, and ϵ-robust, for $\epsilon > 0$, is at best as TCP-friendly as Robust-AIMD(α,β,ϵ) and upper bound the TCP-friendliness of the latter.

Our analysis of Robust-AIMD(α,β,ϵ) reveals that the upper bound of Theorem 3 is tight for $\epsilon << \beta$, in the sense that Robust-AIMD(α,β,ϵ) exactly matches it, giving rise to the following result:

Theorem 4. *For any $\alpha > 0$, $\beta > 0$, and $\epsilon > 0$ such that $\epsilon << \beta$, there exists a protocol that is α-fast-utilizing, β-efficient, ϵ-robust, and $\frac{3(1-\beta)}{(4(\frac{C+\tau}{1-\epsilon})-\alpha)(1+\beta)}$-TCP-friendly.*

We will revisit Robust-AIMD when discussing the Pareto frontier for protocol design in Section 5.2.

4.3 On Establishing TCP-Friendliness

Often, to establish that a congestion control protocol is TCP-friendly, simulation or experimental results are presented to demonstrate that it does not harm TCP by "too much". What, though, guarantees that friendliness towards a certain TCP variant implies friendliness towards other TCP variants? Our next result suggests a principled approach to establishing TCP-friendliness: prove that the protocol is friendly towards a specific TCP variant (say, TCP Reno) and deduce that it is at least as friendly to any "more aggressive" TCP variant. We introduce the following terminology: a protocol P is *more aggressive* than a protocol Q if for any combination of P- and Q-senders, and initial sending rates, from some point in time onwards, the average goodput of *any* P-sender is higher than that of *any* Q-sender.

Theorem 5. *Let P and Q be two protocols such that (1) each protocol is either AIMD, BIN, or MIMD, (2) P is α-TCP-friendly, and (3) Q is more aggressive than Reno. Then, P is α-friendly to Q.*

AIMD and BIN are special cases for BIN [9]. Since BIN protocol converges to a steady state, all BIN protocols can be ordered by their aggressiveness according to the sum $l + k$ in the protocol (where l and k are in the protocol specification)

4.4 Loss-based vs. Latency-Avoiding Protocols

Mo et al. show that the loss-based TCP Reno is very aggressive towards the latency-avoiding TCP Vegas [32]. Intuitively, this is a consequence of Vegas backing off upon exceeding some latency bound while Reno continues to increase its rate since it is oblivious to latency. The following result shows that any "reasonable" loss-based protocol is extremely unfriendly towards *any* member of a broad family of latency-avoiding protocols that includes TCP Vegas.

We say that a protocol is *latency-decreasing* if, for some fixed $\alpha > 0$, when the experienced RTT is not within an α-factor from the minimum possible RTT the sender must decrease its rate. Put formally: if, for some time t, $RTT(t) > \alpha \times 2\Theta$, then $x_i^{(t+1)} < x_i^{(t)}$ for every P-sender i. We point out that TCP Vegas, which documents the minimum experienced RTT and decreases its rate when the experienced RTT is "too high" with respect to that value, falls within this class of protocols (for the appropriate choice of α).

Theorem 6. *For any $\alpha > 0$ and $\beta > 0$, any loss-based protocol that is α-efficient is not β-friendly to any latency-decreasing protocol.*

Proof sketch for Theorem 6: Consider a loss-based protocol P that is α-efficient for some $\alpha > 0$, and a latency-decreasing protocol Q. Suppose that a single P-sender is sharing the link with a single Q-sender. As Q is latency-decreasing, there exists some value δ such that when the experienced RTT exceeds $\delta \times 2\Theta$ the Q-sender must decrease its rate. Now, let the size of the buffer τ be such that the when the sum of sending rates is $\alpha(C + \tau)$ the RTT is strictly higher than $\delta \times 2\Theta$. Consider the scenario that the initial congestion window of P is $C + \tau$ and the initial congestion window of Q is 1 MSS, which is assumed to be negligible with respect to $C + \tau$. This initial choice of congestion window sizes captures a scenario in which P-sender is fully utilizing the buffer when the Q-sender starts sending (or, alternatively, the Q-sender experienced a timeout while the P-sender sends at a high rate). Our proof shows that henceforth, the congestion window of the P-sender will always be lower bounded by $\alpha(C+\tau)$ and so the RTT will always be at least $\delta \times 2\Theta$. Consequently, the Q-sender cannot increase its congestion window and must send at its minimum rate. To see why, observe that as P is loss-based, from the P-sender's perspective, this environment is indistinguishable from the environment in which the link capacity is $C + \tau$ and there is no buffer. In addition, so long as the Q-sender sends at a fixed (its minimum) rate, the P-sender cannot distinguish between the (actual) scenario that the link is shared with others and the scenario that the P-sender is alone on the link. Consequently, α-efficiency dictates that the congestion window of the P-sender remain of size at least $\alpha(C + \tau)$. This, in turn, by our choice of τ forces the Q-sender to not increase its congestion window size. The theorem follows.

4.5 When Can Axioms Coexist?

Our axiomatic derivations above establish that certain desiderata *cannot* be simultaneously achieved by *loss-based* protocols (see Claim 1, Theorem 2, and Theorem 3). Importantly, these impossibility results hold regardless of the number of senders and network parameters (bandwidth, buffer size, etc.). We show below that, in certain conditions, *latency-sensitive* protocols can bypass the impossibility results of this section and, in fact, well-satisfy *all* of the requirements of Section 3. However, as the discussion below illustrates, this is subtle and, in particular, depends on the network parameters. Thus, going beyond the impossibility results of this section requires not

only considering congestion signals other than loss (latency) but also a more nuanced, parameter-dependent, discussion.

To see this, consider the following protocol design, termed "AIMD$_l$". Intuitively, AIMD$_l$ is the latency-sensitive extension of AIMD (see Section 2). AIMD$_l$ gradually increases its rate so long as no loss is experienced and its experienced latency does not increase. In the event that loss is experienced, or the RTT increases, AIMD$_l$ decreases its rate. Specifically:

AIMD$_l$(a,b):

$$x_i^{(t+1)} = \begin{cases} x_i^{(t)} + a & \text{if } L^{(t)} = 0 \text{ or } RTT(t) \leq RTT(t-1) \\ x_i^{(t)} b & \text{if } L^{(t)} > 0 \text{ or } RTT(t) > RTT(t-1) \end{cases}$$

Consider first the scenario that the buffer size τ is very large. Intuitively, in this scenario, the dynamics of congestion control are precisely as with AIMD(a,b)-senders on a link with no buffer, as every time traffic enters the buffer the reactions of the AIMD$_l$ senders is identical to that of AIMD senders experiencing loss. Consequently, in terms of efficiency, fairness, convergence, and fast-utilization, AIMD$_l$(a,b) exhibits the exact same guarantees as AIMD(a,b) (to be presented in Table 1 of Section 5).

However, when the buffer size is very large, AIMD$_l$ can easily be shown to be 0-loss avoiding (as it shies away from increased latency and so empties the buffer). In addition, AIMD$_l$ is very TCP-friendly. In fact, being latency-decreasing (see Section 4), Theorem 6 applies to AIMD$_l$ and so the loss-based TCP Reno is actually not friendly towards AIMD$_l$. Thus, under these conditions, AIMD$_l$ provides lower loss-avoidance (by Claim 1) and higher TCP-friendliness (by Theorem 6) than achievable by *any* loss-based protocol.

Moreover, for "good" choices of a and b, say, $a = 1$ and $b = 0.99$, *AIMD$_l$* can achieve "good scores" in all these metrics but robustness. In fact, the analogously defined latency-sensitive variant Robust-AIMD$_l$ of Robust-AIMD attains fairly high scored with respect to *all* eight requirements of Section 3.

Observe, however, that when no buffer exists, or the buffer size τ is small, AIMD$_l$ and Robust-AIMD$_l$, now being effectively loss-based only, are reduced to AIMD and Robust-AIMD, respectively. Consequently, these protocols are no longer $0 - loss$, nor very TCP-friendly, in light of the impossibility results of Claim 1, and Theorem 2, and Theorem 3.

5 Protocol Analysis and Design

Our theoretical framework, as presented in Section 2, allows us to associate each congestion control protocol with a 8-tuple of real numbers, representing its scores in the eight metrics. We present below our results in this direction and explain the implications for protocol design.

5.1 Mapping Protocols to Points

Table 1 presents our theoretical results mapping protocols to points in our 8-dimensional metric space for the families of protocols described in Section 2 (i.e., AIMD, MIMD, BIN, CUBIC), and also for ROBUST-AIMD (discussed in Section 4).

We present in angle brackets *worst-case bounds* across all choices of network parameters (e.g., very shallow buffer, very high number of senders, etc.). To gain insight into how the specific link parameters (capacity C, buffer size τ) and number of senders n affect efficiency, stability, etc., we complement some of the worst-case bounds with more nuanced results reflecting the dependence on these parameters.

The results regarding the fairness and TCP-friendliness of BIN are derived from [9] and the fairness of CUBIC is derived from [24]. Observe that MIMD is ∞-fast-utilizing as its rate increases superlinearly. As all protocols considered are loss-based, their scores for latency avoidance are

Protocol	Efficiency	Loss-Avoiding	Fast-Utilizing	TCP-Friendly	F
AIMD(a,b)	$\min(1, b(1+\frac{\tau}{C}))$ <b>	$1-\frac{C+\tau}{C+\tau+na}$ <1>	<a>	$<\frac{3(1-b)}{a(1+b)}>$	<
MIMD(a,b)	$\min(1, b(1+\frac{\tau}{C}))$ <b>	$<\frac{a}{1+a}>$	<∞>	$\frac{2log_a \frac{1}{b}}{C+\tau-2log_a \frac{1}{b}}$ <0>	<
BIN(a,b,l,k)	$\min(1, (1-b)(1+\frac{\tau}{C}))$ <(1 - b)>	$1-\frac{C+\tau}{C+\tau+na(\frac{C+\tau}{n})^k}$ <1>	<a> if k=0 <0> if k>0	$<\sqrt{\frac{3}{2}}(\frac{b}{a})^{\frac{1}{1+l+k}}>$ if $l+k \geq 1$ <0> otherwise	<
CUBIC(c,b)	$\min(1, b(1+\frac{\tau}{C}))$ <b>	$1-\frac{C+\tau}{C+\tau+nc}$ <1>	<c>	$\sqrt{\frac{3}{2}}\sqrt[4]{\frac{4(1-b)}{c(3+b)(C+\tau)}}$ <0>	<
Robust-AIMD(a,b,k)	$\min(1, \frac{b(1+\frac{\tau}{C})}{1-k})$ $<\frac{b}{1-k}>$	$\frac{(C+\tau)k+na(1-k)}{(C+\tau)+na(1-k)}$ <1>	<a>	$\frac{3(1-b)}{(4(\frac{C+\tau}{1-k})-a)(1+b)}$ <0>	<

Table 1. Protocol Characterization. Worst-case bounds shown within angle-brackets.

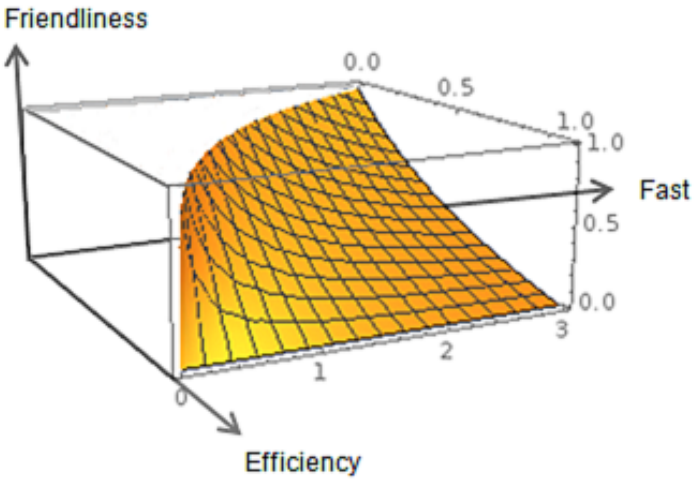


Fig. 1. Pareto frontier of Efficiency, Friendliness, and Fast-utilization

unbounded and so omitted from the table. Also omitted are our results for robustness: all protocols are 0-robust, with the exception of $Robust-AIMD(a,b,k)$, which is easy to show is k-robust.

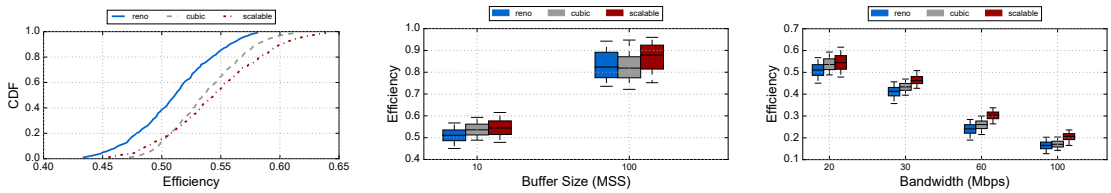
Our proofs of the bounds in the entries of Table 1 are derived by examining the relevant protocol's dynamics and are provided in Appendix B.

5.2 Seeking Points on the Pareto Frontier

The Pareto frontier for protocol design. Not every point in the 8-dimensional space induced by our metrics is *feasible*, in the sense that there are some points such that no protocol can attain their associated scores. Indeed, by Theorem 2, no loss-based protocol can *simultaneously* achieve near-perfect scores for fast-utilization, efficiency, *and* TCP-friendliness. We refer to the region of *feasible points* in our 8-dimensional space as the *feasibility region for protocol design*. Our focus is on the *Pareto frontier* of this feasibility region. A feasible point is on the Pareto frontier if no other

feasible point is strictly better in terms of one of our metrics without being strictly worse in terms of another metric. Any point on the Pareto frontier thus captures scores that are both attainable by a congestion control protocol and cannot be strictly improved upon. Different points on this frontier capture different tradeoffs between our metrics.

To illustrate the above, let us restrict our attention to the efficiency, fast-utilization, and TCP-friendliness metrics and consider the tension between these metrics, as captured by Theorem 2. Figure 1 describes the Pareto frontier in the 3-dimensional subspace spanned by these three metrics. Points on this Pareto frontier are of the form $(\alpha, \beta, \frac{3(1-\beta)}{\alpha(1+\beta)})$ (corresponding to fast-utilization, efficiency, and TCP-friendliness scores, respectively). Observe that each of these points is indeed feasible as AIMD(α,β) attains these scores (see Table 1). We now add the requirement of robustness to non-congestion loss to the mix. Now, Robust-AIMD, presented in Section 4, cannot be improved upon, in terms of one of the four metrics under consideration, without lowering its score in another (by the results in Table 1 and Theorem 3), and thus lies on the Pareto frontier of their induced 4-dimensional space. We view the above as examples of how the axiomatic approach can be leveraged to identify points in the design space that lie on the Pareto frontier, and guide the theoretical investigation and experimental exploration of such points.



(a) (b) (c)

Fig. 2. The figures show the efficiency values achieved for Reno, Cubic and Scalable TCP. Figure (a) shows the complete CDF for four senders sharing a 20Mbps link with buffer of size 10MSS. Figures (b) and (c) show the box plots (with 5th, 25th, 50th, 75th, and 95th percentile values), as the buffer size and bandwidth are varied from the default setting in Figure (a).

(n,BW)	(2,20)	(2,30)	(2,60)	(2,100)	(3,20)	(3,30)	(3,60)	(3,100)	(4,20)	(4,30)	(4,60)
Cubic	0.281	0.516	0.528	0.497	0.505	0.664	0.625	0.577	0.615	0.721	0.665
Scalable	0.218	0.285	0.484	0.300	0.350	0.312	0.370	0.379	0.316	0.278	0.325

Table 2. TCP-Friendliness comparison between Cubic and Scalable

5.3 Experimental Validation

We validate our theoretical findings (shown in Table 1) via experiments with the Emulab network emulator [48]. We stress that the purpose of our experimental investigation is not to show that the achieved efficiency, convergence, friendliness, etc., exactly match the theoretical bounds in Table 1, since differences would naturally arise from real-world aspects not captured by our theoretical framework (such as timeouts). Nonetheless, we show how in spite of its simplifying assumptions, our theoretical analysis successfully captures the *trends* produced by real-world interactions between congestion control protocols.

Experimental framework. We experiment with the following protocols, implemented in the Linux kernel: TCP Reno, TCP Cubic, and TCP Scalable. Our experiments investigated the interaction of a varying number of connections (2-4) on a single link, for varying bandwidths (20Mbps, 30Mbps,

60Mbps, and 100Mbps), varying buffer sizes (10 MSS / 100 MSS), and a fixed RTT of 42ms. We point out that each of the evaluated protocols represents a family of protocols formulated in Section 2: Reno is AIMD(1,0.5), TCP Cubic is CUBIC(0.4,0.8), Scalable is MIMD(1.01,0.875) in some environments and AIMD(1,0.875) in others (e.g., when RTT=42ms and the buffer size is 100MSS, Scalable is AIMD for BW$\leq$43.5Mbps and MIMD for bandwidth BW$\geq$43.5Mbits/sec). Each experiment lasted 2.5 minutes and iperf was used to measure the throughput attained by each sender at 0.5s intervals.

Efficiency. To see how the trends in our theoretical results translate to real-world behavior, we first consider the scenario with four senders where the buffer size is 10MSS and the bandwidth is 20Mbps. This, by the results in Table 1, induces a very clear hierarchy of protocols (from "worst" to "best"): Reno (0.57), Cubic (0.91), Scalable (1.0). Fig 2(a) plots CDF of the efficiency observed in our experiments, computed as the sum of rates for each sender averaged over 0.5s intervals, divided by the link bandwidth. A point (x, y) in such a CDF captures the fraction of time-intervals (y) over the entire experiment duration in which the efficiency was less than x.[3] These experimental results indeed reveal the exact same hierarchy over protocols. Figure 2(b) and (c) further show the trends as we vary the buffer size and bandwidth respectively. Increasing the buffer size, keeping the bandwidth fixed, increases the $\frac{\tau}{C}$ ratio, thus increasing efficiency value (given by $b\left(1+\frac{\tau}{C}\right)$). On the other hand, increasing the bandwidth, keeping the buffer size fixed, decreases the $\frac{\tau}{C}$ ratio, thus decreasing efficiency.

TCP-Friendliness. Table 2 presents a side-by-side comparison of the TCP-friendliness of the examined protocols across multiple choices of number of senders n and link bandwidths BW for a buffer of size 100MSS (see first row). Each experiment measured the average sending rate $\bar{x}$ of $n-1$ senders that employ the protocol under consideration, competing with a single TCP-Reno sender on a single link. The level of TCP-friendliness was then set to be $\frac{x^{Reno}}{\bar{x}}$, where x^{Reno} is the average sending rate of the Reno sender. Our empirical results for TCP-friendliness present in Table 2 adhere to the theoretical lower-bound in Table 1. The theoretical lower-bound on α-friendliness of Cubic in our setting is computed to be between 0.22 and 0.28. The empirical friendliness value is between 0.28 and 0.72. The difference here primarily arises from a conservative estimate of loss rate used in our worst-case theoretical analysis of Cubic friendliness. Observe, however that in some of the empirical settings Cubic's friendliness to TCP is very close to this lower-bound. Scalable mostly behaves as AIMD(1,0.875) as its rate does not exceeds 44Mbps frequently, and so according to Table 1 its TCP-friendliness level is close to 0.2, which is close to what we see in our evaluated scenarios. Our results for other choices of buffer sizes are also consistent with the theoretical results.

Our theoretical results for Robust-AIMD suggest that it constitutes a point in the design space of congestion control protocols that provides performance comparable to AIMD in a significantly more robust manner, while being more friendly to legacy TCP connections than PCC (whose behavior is strictly more aggressive than MIMD(1.01,0.99)). In fact, the proof of our theoretical result regarding the TCP-friendliness of Robust-AIMD shows that its TCP-friendliness is *monotone* in the number of Robust-AIMD connections, in the sense that the more Robust-AIMD connections share a link the better its friendliness to TCP connections on the link becomes.

[3]Recall that α-efficiency of a protocol P means that from some point in time onwards any number of competing senders that employ P will utilize at least an α-fraction of the link. Hence, even a protocol that achieves over 95% link utilization 99.999% of the time but occasionally drops to 50% link-utilization is only formally $\frac{1}{2}$-efficient. Thus, the notion of α-efficiency, while simple to reason about, is very sensitive to "noise". Consequently, phenomena that are not modeled in our framework, such as occasional transition to slow-start as a result of a timeout, can lead to very low efficiency according to our metric, even if the protocol exhibits very high link-utilization in practice.

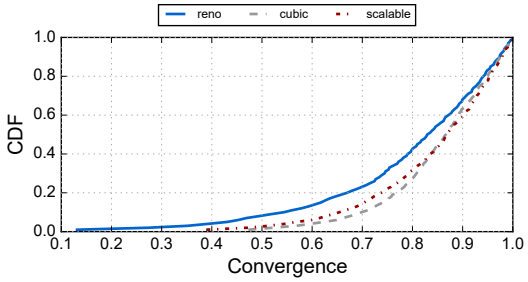


Fig. 3. CDF showing convergence for Reno, Cubic and Scalable TCP for four senders on 20Mbps link with a buffer of size10MSS.

(n,BW)	(2,20)	(2,30)	(2,60)	(2,100)	(3,20)	(3,30)	(3,60)	(3,100)	(4,20)	(4,30)
R-AIMD / PCC	2.48x	1.69x	1.19x	1.94x	1.76x	1.35x	2.75x	1.92x	1.30x	2.47x

Table 3. TCP-friendliness of Robust-AIMD(1,0.8,0.01) vs. PCC

We experimentally evaluated Robust-AIMD(1,0.8, ϵ), for ϵ values of 0.005, 0.007, and 0.01 (corresponding to loss rates of 0.5%, 0.7%, and 1%, respectively). Representative experimental results comparing Robust-AIMD's TCP friendliness to PCC's TCP friendliness appear in Table 3. Each entry in the table specifies the improvement in % of Robust-AIMD(1,0.8) over PCC for different choices of number of senders on the link (n) and link bandwidth, constant RTT of 42ms and buffer size of 100 MSS. Observe that Robust-AIMD consistently attains >1.5x TCP-friendliness than PCC (1.92x improvement on average).

Convergence. Our theoretical results in Table 1 establish the following ordering of protocols, in terms of α-convergence (from "worst" to "best"): Reno (0.67), Cubic (0.89), Scalable (0.93). Figure 3 shows the CDF of convergence for our default scenario with four senders using a 20Mbps link with buffer sized at 10MSS, generated as follows: Consider a sender i competing with other senders over the bandwidth of a single link. Let $\bar{s_i}$ denote the sender's average rate over the entire duration of the experiment. A point (x, y) in the plot captures the fraction (y) of 0.5s-long time-intervals over the entire duration and across different senders, for which the convergence value was less than x. Convergence value for a time-interval i was computed as $\frac{s_i}{\bar{s_i}}$ for $s_i < \bar{s_i}$ and $\left(2 - \frac{s_i}{\bar{s_i}}\right)$ for $s_i > \bar{s_i}$, where s_i was the average rate in each interval. Thus, the CDF captures the distance of a sender's rate from the fixed rate $\bar{s_i}$, as defined in §3. Our experimental results reflect the same general hierarchy of protocols as our theoretical results, with Reno's convergence being smaller than Cubic and Scalable and the last two having similar convergence behavior. We saw similar trends in other experimental settings.

Fairness. As presented in Table 1 all evaluated protocols are, in theory, 1-fair (as Scalable behaves like AIMD in the environments we tested). Our experimental results have been tabulated in Table 4. Specifically, Table 4 (a) presents the results of experiments for $n = 2 - 4$ senders when all senders start sending at the same time. Table 4 (b) presents results for the same experimental conditions, but where one of senders starts 100s after others. We find that, for both cases, all protocols indeed come fairly close to perfect fairness. Our results for other choices of parameters are also consistent with the theoretical results.

Robustness. We now turn our attention to the robustness of protocols under different random loss rates. Our theoretical result regarding the robustness of Robust-AIMD(1,0.8, ϵ) establishes that

(n,BW)	(2,100)	(3,100)	(4,100)
Reno	0.931	0.824	0.968
Cubic	0.858	0.878	0.926
Scalable	0.636	0.853	0.961

(a) All senders start at same time.

(n,BW)	(2,100)	(3,100)	(4,100)
Reno	0.921	0.898	0.952
Cubic	0.913	0.952	0.883
Scalable	0.944	0.911	0.967

(b) One sender starts late (after 100s)

Table 4. Fairness of Reno, Cubic and Scalable TCP for 100Mbps bandwidth and 100MSS buffers

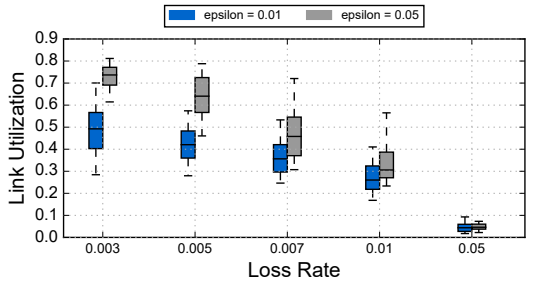


Fig. 4. Box-plots (with 5th, 25th, 50th, 75th, and 95th percentile values) comparing robustness of Robust-AIMD for $\epsilon = 0.01$ and $\epsilon = 0.05$

the higher the value of ϵ the more robust the protocol is. We experimentally evaluated Robust-AIMD(1,0.8, ϵ), for ϵ values of 0.01 and 0.05 (corresponding to loss rates of 1% and 5%, respectively). Fig. 4 presents box plots of link utilization (sending rate divided by the bandwidth) for a single sender using a link of capacity 100Mbps with buffer size 100MSS under varying loss rates. Our experimental results reflect the expected hierarchy over Robust-AIMD protocols: the sender with the higher $\epsilon = 0.05$ is more robust than the sender with $\epsilon = 0.01$, with the difference between the two decreasing as the loss rate is increased. Our results for other choices of parameters are also consistent with the theoretical results.

6 High Throughput or Low Latency/Loss? Pick!

As shown in Section 4 and in Section 5, the simple fluid-flow model of dynamics of congestion control protocols presented in Section 2 enables reasoning about fundamental tradeoffs in protocol design that, we believe, extend well beyond the scope of the model. However, the limitations of this simplified model render it impossible to reason about some other fundamental tradeoffs. We thus extend this model to establish another fundamental tradeoff—between the desideratum of achieving high (average) throughput and the desideratum of maintaining low (average) latency/loss in unpredictable and dynamic environments.

Wireless networks have been the subject of much attention as the unpredictability and variability of available bandwidth in such networks make them particularly challenging from a protocol design perspective. The experimental results of recent studies (see Figures 9 and 10 in [50] and Figure 6 in [19]) suggest that *simultaneously* attaining high throughput *and* keeping latency low in such environments is impossible, and that different protocols reflect different tradeoffs between these two desiderata. We show how this can be formalized and proved within our axiomatic framework.

6.1 Revisiting Our Model and Axioms

Revised model. We change the model of Section 2 as follows. Now, the link's bandwidth can change every RTT. Each sender must select a rate for time step t at the beginning of the time step and without knowledge of the bandwidth of the link $B^{(t)}$ at that time step. We assume that for all t, $B_1 \leq B^{(t)} \leq B_2$ for some fixed values B_1 and B_2. Let the capacity $C^{(t)}$ of the link at time t be $C^{(t)} = B^{(t)} \times 2\Theta$ (recall that Θ is the link's propagation delay). All other aspects of the model remain as in Section 2. We show below that well-utilizing the link in this model is at odds with maintaining low latency and loss even if only a *single* sender is using the link. Hence, to simplify exposition, let us focus henceforth on the single-sender scenario. Our theoretical results below easily extend to multiple senders.

We note that the above extension to our model is inspired by the experimental framework for evaluating congestion control protocols on LTE networks used in [19, 50] (using Mahimahi [34] on Verizon-LTE traces).

Revised axioms. To reason about the tradeoffs between different desiderata, we revisit the efficiency, loss-avoidance, and latency-avoidance axioms from Section 3 (Metrics I, III, and VIII, respectively), and adapt the formulation of these axioms to the above extended model. Specifically, the new formulations of the axioms capture not only *eventual* behavior ("from some time onwards") but also temporal aspects of congestion control protocols (how well the protocol behaves within a certain time frame).

The new formulation of the efficiency axiom quantifies how close the average utilization of the link within a time frame is to the maximum volume of traffic the link could have carried during that time. This enables us to reason about the average link utilization over time. As the link's RTT can vary over time, the efficiency axiom formulation averages over link utilization at different time steps proportionally to the RTTs of these time steps.

Efficiency. We say that a congestion-control protocol P is *α-efficient* if when the sender employs P, then for any time T,

$$\frac{\Sigma_{t=1}^{T} RTT(t) \times \frac{\min(X^{(t)}, C^{(t)})}{C^{(t)}}}{\Sigma_{t=1}^{T} RTT(t)} \geq \alpha$$

Our next two axioms bound the average *per-packet* loss rate and latency.

Loss-avoidance. We say that a congestion-control protocol P is *α-loss-avoiding* if when the sender employs P, then for any time T,

$$\frac{\Sigma_{t=1}^{T} X^{(t)} \times L^{(t)}}{\Sigma_{t=1}^{T} X^{(t)}} \leq \alpha.$$

Latency-avoidance. We say that a congestion-control protocol P is *α-latency-avoiding* if when the sender employs P, then for any time T, for sufficiently large buffer size τ,

$$\frac{\Sigma_{t=1}^{T} X^{(t)} \times RTT(t)}{\Sigma_{t=1}^{T} X^{(t)}} < (1 + \alpha)2\Theta.$$

6.2 Throughtput *vs.* Latency

Suppose, for now, that the buffer is of infinite size so as to remove packet loss from consideration and focus solely on the implications of rate choices for latency. As discussed next, intuitively, attaining high throughput and maintaining low latency in this setting cannot be accomplished simultaneously. Clearly, the sender can simply bombard the link with traffic so as to achieve consistently high throughput at the cost of high latency. Alternatively, the sender can send at very low rates to attain perfect latency at the cost of low average bandwidth utilization. Can congestion control

protocols achieve both desiderata *simultaneously*? Our next two results show that the answer to this question is negative. This sheds light on the recent experimental findings in [19, 50] for LTE networks (see Figures 9 and 10 in [50] and Figure 6 in [19]), which experimentally demonstrate the tradeoff between the two desiderata.

Observe that the protocol that leaves the congestion window fixed at $2\theta B_1$ (the minimum possible bandwidth of the link) is trivially $\frac{B_1}{B_2}$-efficient (as the maximum link bandwidth is B_2) and minimizes latency (i.e., is 0-latency avoiding). We show next that any improvement in efficiency over this simple protocol comes at the cost of worse latency, where the higher the improvement in efficiency is the worse the resulting latency is.

THEOREM 7. *Any protocol that is α-efficient for $\alpha > \frac{B_1}{B_2}$ is not $\frac{B_1(B_2-B_1)+\left(\gamma-(\gamma)^2\right)B_2^2-2B_2\sqrt{(1-\gamma)\gamma B_1(B_2-B_1)}}{((1-\gamma)B_2-B_1)^2}$-latency-avoiding for any $\gamma < \alpha$.*

We next present the "converse" of Theorem 7, which shows that maintaining low latency must come at the cost of suboptimal bandwidth utilization.

THEOREM 8. *Any protocol that is α-latency avoiding for $\alpha > 0$, is not $\frac{(\gamma+1)(B_2-B_1)+2B_1-B_2+2\sqrt{B_1(B_2-B_1)\gamma}}{B_2(\gamma+1)}$-efficient for any $\gamma > \alpha$.*

We now provide a sketch of the proof of Theorem 7. The proofs of the other theorems of this section employ a similar methodology.

Proof sketch for Theorem 7. To provide the intuition for the proof and simplify exposition, consider the following setting: The single sender participates in a game against an adversary; at the beginning of each time step t, the sender sets the size of its congestion window and then the adversary selects the bandwidth $B^{(t)}$ for that time step. We will show that link bandwidths can be selected in a manner that induces average latency of at least $\frac{2\theta B_2\left((1-\alpha)(B_2-B_1)+\alpha B_1-2\sqrt{(1-\alpha)\alpha B_1(B_2-B_1)}\right)}{((1-\alpha)B_2-B_1)^2}$, yielding the bound in the statement of the theorem. While this setting reflects a view of the environment as "hostile" and capable of adapting the link's bandwidth in response to the sender's actions, the bound of Theorem 7 can be established even in model where the link's bandwidth at each point in time is drawn from a *fixed* probability distribution over B_1 and B_2 that is constant across time (and so the environment is completely oblivious to the sender's choices of rates and exhibits stochastic regularity). We later discuss how the proof methodology presented below (for the adversarial setting) can be extended to the stochastic setting.

Consider the following strategy for the adversary: Choose a fixed "threshold" $\frac{B_1}{B_2} < \beta < \alpha$. When the sender's chosen window size is at most $\beta B_2 \times 2\Theta$, set B_2 as the link bandwidth for that time step. When the sender's chosen window size is above $\beta B_2 \times 2\Theta$, set B_1 as the link bandwidth for that time step.

What should the sender do against this adversarial strategy? Observe that as the link's bandwidth is always within the interval $[B_1, B_2]$, the sender cannot gain anything, in terms of performance, from choosing a congestion window of size below $B_1 \times 2\Theta$ or above $B_2 \times 2\Theta$. In fact, the following is easy to show.

CLAIM 2. *The sender's optimal strategy always selects congestion window sizes within the interval $[\beta B_2 \times 2\Theta, (\beta+\epsilon)B_2 \times 2\Theta]$ for an arbitrarily small choice of $\epsilon > 0$.*

To see this, consider first the scenario that the sender's congestion window is strictly beneath $\beta B_2 \times 2\Theta$. Then, as in this scenario the realized bandwidth is B_2, a congestion window of size $\beta B_2 \times 2\Theta$ would have led to the exact same realized link bandwidth, but would have resulted in strictly better link utilization (and the same latency). Now, consider the scenario that the sender's

congestion window is strictly above $(\beta + \epsilon)B_2 \times 2\Theta$. Then, as in this scenario the realized link bandwidth is B_1, a congestion window of size precisely $(\beta + \epsilon)B_2 \times 2\Theta$ would have led to the exact same realized link bandwidth, but would have resulted in strictly better latency (and the same link utilization, i.e., 100%). Claim 2 implies that the sender's best strategy is effectively constantly sending at a rate of $\beta B_2 \times 2\Theta$, and its decisions effectively boil down to whether the congestion window should be of size precisely $\beta B_2 \times 2\Theta$, which we will refer to as deciding "DOWN", or the congestion window size should be slightly above $\beta B_2 \times 2\Theta$, which we will refer to as deciding "UP". Given the adversary's strategy, every time UP is decided the realized link bandwidth is B_1 and every time DOWN is decided the realized bandwidth is B_2.

Since the sender is α-efficient on average, no matter what the link bandwidths are, at least an α-fraction of the link must be utilized on average. This implies that the sender cannot decide DOWN "too often" as this will result in "too many" time steps in which the sender's window size is $\beta B_2 \times 2\Theta$ but the link capacity is $B_2 \times 2\Theta$ and, as $\beta < \alpha$, the link utilization is "low". The following claim formalizes this intuition.

Claim 3. *For any $0 < \alpha < 1$ and $\frac{B_1}{B_2} < \beta < \alpha$, the sender's optimal strategy cannot decide DOWN in more than a $\frac{(1-\alpha)\beta B_2}{(1-\alpha)\beta B_2+(\alpha-\beta)B_1}$-fraction of the time steps.*

This, of course, implies that the sender's optimal strategy must decide UP in at least (in fact, precisely) a $\frac{(\alpha-\beta)B_1}{(1-\alpha)\beta B_2+(\alpha-\beta)B_1}$-fraction of the time steps. Since every time UP is decided, the realized bandwidth is B_1, this implies that in $\frac{(\alpha-\beta)B_1}{(1-\alpha)\beta B_2+(\alpha-\beta)B_1}$-fraction of the time steps, the sender sends at a rate that is higher than the bandwidth and so suffers from increased latency. Specifically, at each such time step t more than $\beta B_2 \times 2\Theta$ packets are sent, and so the latency for that time step $RTT(t)$ is at least $2\theta \times (1 + \frac{\beta B_2 - B_1}{B_1})$. Even assuming that in all other time steps the latency is 2Θ (the minimum), this results in average latency of $\frac{2\theta(1-\beta)\beta B_2}{(\alpha-\beta)B_1+\beta(1-\alpha)B_2}$.

Given the sender's optimal strategy with respect to a value β, as discussed above, the adversary can compute the value of β that maximizes the sender's average latency. Our calculations show that the resulting average latency value cannot be within a multiplicative factor of

$$1 + \left[\frac{B_1(B_2 - B_1) + \left(\alpha - \alpha^2\right) B_2^2 - 2B_2\sqrt{(1-\alpha)\alpha B_1(B_2 - B_1)}}{((1-\alpha)B_2 - B_1)^2} \right]$$

of the minimum latency (2Θ). As the above formula is monotone increasing in α for $\alpha \in [\frac{B_1}{B_2}, 1]$, this yields the bound of Theorem 7.

While the proof sketch above considers an adversarial environment, the bound of Theorem 7 applies also when the link's bandwidth is B_1 with probability p and B_2 with probability $1 - p$ for a *fixed* probability p that is constant over time. Extending our proof technique to this context involves setting $p = \frac{(\alpha-\beta)B_1}{(1-\alpha)\beta B_2+(\alpha-\beta)B_1}$ (i.e., precisely the fraction of time presented in Claim 3). Then, the arguments above can be adapted to show that the sender's optimal strategy is setting the congestion window size to be precisely $\beta B_2 \times 2\Theta$, leading to the same lower bound on latency. More specifically, our proof shows that under this probability distribution over link bandwidth, the sender's optimal strategy is constantly setting its window size to $\beta B_2 \times 2\Theta$. The calculation of the average latency resulting from this choice of congestion window yields the bound of Theorem 7.

6.3 Throughput *vs.* Loss

As the next two results show, when the buffer size is finite, the desideratum of attaining high throughput and the desideratum of keeping (average) loss rate low are also at odds.

Theorem 9. *Any protocol that is α-efficient for $\alpha > \frac{B_1}{B_2}$ is not $\frac{(2-\gamma)B_2-B_1-2\sqrt{(1-\gamma)B_2(B_2-B_1)}}{\gamma B_2}$-loss avoiding, for all $\gamma < \alpha$.*

Theorem 10. *Any protocol that is α-latency avoiding for $\alpha > 0$, is not $\frac{2\sqrt{\gamma(B_2-B_1)(B_1+B_2\gamma)}-B_1\gamma+B_1+2B_2\gamma}{B_2(\gamma+1)^2}$-efficient for all $\gamma > \alpha$.*

7 Related Work

The axiomatic approach. The axiomatic approach is deeply rooted in the theory of social choice, whose focus is on how to combine individual opinions into collective decisions. Classical applications include Arrow's celebrated Impossibility Theorem [7] and the Gibbard-Satterthwaite Theorem [22, 41].

The axiomatic approach has also been applied to many computer science environments, including ranking systems [2, 3], reputation systems [47], recommendation systems [6], routing protocols [28], link prediction [17], sustainable developments [15], and relating ACK-clocking and actual dataflow rate [12]. We are, to the best of our knowledge, the first to apply the axiomatic approach to congestion control protocols.

Network-utility maximization (NUM). NUM is a prominent framework for analyzing and designing TCP protocols, pioneered by Kelly et al. [26]. [33] shows how a simple formulation of a convex program for maximizing the "social welfare" of senders of traffic across a network can be used to derive both congestion control policies for the end-hosts and queueing policies for in-network devices (routers) via the primal dual schema. NUM is useful for both (1) gaining insights into *existing* (TCP) protocols (by reverse engineering the implicit utility functions being optimized) and (2) designing new TCP protocols (by building into the protocol the desired utility function) with respect to specific, perdetermined objectives (captured by connections' utility functions). Our focus, in contrast, is on outlining the possibility/impossibility borderline for congestion control by studying the delicate interplay between different desired properties.

Control-theoretic approaches to TCP congestion control. Control theory has been applied to reason about the implications of rate modulation by a *specific* protocol for performance, stability, and fairness [43], with respect to different active queue management schemes [25, 38], buffer sizes [39], network topologies and traffic patterns, etc. As discussed above, our focus is on the inherent limitations for congestion control protocol design.

Analyzing TCP dynamics. Our model for reasoning about dynamics of congestion control builds upon classical models for analyzing TCP dynamics [9, 13, 16, 30, 35, 36, 40].

8 Conclusion

We have put forth an axiomatic approach for analyzing congestion control designs, which complements the current simulation/experimentation and theoretical approaches. Our approach uses a simple model, and a set of axioms, to gain insight into which properties can co-exist, and which follow from other properties. Our results characterize a Pareto frontier for congestion control that defines the borderline between possibility and impossibility. While derived in the context of a simple model, these results shed light on past empirical and experimental findings and provide guidance for future design efforts. We view our results as a first step and leave the reader with many research directions relating to the extension of our model and the re-examination of our axioms/metrics (Section 3). Thus far, axiomatic approaches have been applied to a few, very specific, networking environments [17, 28, 42]. We believe that applying the axiomatic approach to other networking contexts (e.g., intradomain [28] and interdomain routing, traffic engineering,

in-network queueing [45], network security) could contribute to more principled discussions about these contexts.

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A Axiomatic Derivation Proofs (Section 4)

Proof of Theorem 2: Let P be loss based, α-fast-utilizing and β-efficient protocol. We next show that P is no more TCP-friendly than AIMD(α, β). Consider first the case of two senders, namely a single P-sender and a single TCP-sender.

- *α-fast-utilization* implies that any P-sender, on average, additively increase its window size by at least α MSS per RTT so long as no packet loss occurs.
- The link-utilization requirement (Metric I) and Lemma 12 (in §A.1), imply that the P sender does not decrease its window size by a multiplicative factor larger than β upon encountering loss.

Combined, this implies that the time duration between two consecutive loss events when competing with TCP Reno (i.e., AIMD($1, \frac{1}{2}$)) on a link is no higher than the time duration between two consecutive loss events when AIMD(α, β) competes with TCP Reno on the same link. The above arguments can be generalized to any m_1 TCP senders competing with m_2 P-senders on a bottleneck link. Now, the sum of window sizes of all senders additively increases by at least $m_1 + m_2\alpha$ when

no loss occurs, and decreases by no more than β when loss does occur. Using arguments similar to those above, we get that the TCP-friendliness of P is no higher than the TCP-friendliness of AIMD(α, β).

To complete the proof, we show that AIMD(α, β) is $\frac{3(1-\beta)}{\alpha(1+\beta)}$ friendly. Suppose that there are m_1 TCP senders, and m_2 P-senders. W.l.o.g., we can treat the m_1 TCP-senders as a single AIMD($m_1, \frac{1}{2}$) sender (denoted by T^*), and the m_2 AIMD(α, β) senders as a single AIMD($\alpha m_2, \beta$) sender (denoted by P^*). By Lemma 11, the TCP window size when competing against (α, β)-AIMD, when the link is fully utilized is

$$x^*_{TCP} = \frac{1}{m_1} T^* = \frac{1}{m_1} \left(\frac{2m_1(1-\beta)(C+\tau)}{2m_1(1-\beta) + \alpha m_2} \right) \tag{6}$$

Once packet loss occurs, the window size of each TCP sender is decreased to $0.5(x^*_{TCP})$ (where after sufficiently long time all TCP senders have equal window size [16]). Hence, the average window size of each TCP sender is $\bar{x}_{TCP} = \frac{1+0.5}{2} x^*_{TCP}$. This yields

$$\bar{x}_{TCP} = \frac{3(1-\beta)(C+\tau)}{4m_1(1-\beta) + 2\alpha m_2} \tag{7}$$

By Lemma 11, the window size of each P-sender when the link is fully utilized is

$$x^*_P = \frac{1}{m_2} P^* = \frac{1}{m_2} \left(\frac{\alpha m_2 (C+\tau)}{2m_1(1-\beta) + \alpha m_2} \right) \tag{8}$$

Upon experiencing loss, the window size of any P-sender is $\beta(x^*_P)$. Hence, the average window size of a P-sender is $\bar{x}_P = \frac{1+\beta}{2} x^*_P$. This yields

$$\bar{x}_P = \frac{\alpha(1+\beta)(C+\tau)}{4m_1(1-\beta) + 2\alpha m_2} \tag{9}$$

Since $\frac{\bar{x}_{TCP}}{\bar{x}_P} = \frac{3(1-\beta)}{\alpha(1+\beta)}$, P is at most $\frac{3(1-\beta)}{\alpha(1+\beta)}$-TCP friendly.

Proof of Theorem 3: Let P be a loss based, α-fast-utilizing, β-efficient, and ϵ-robust protocol. Arguments similar to those in the proof of Theorem 2 imply that to upper bound P's TCP-friendliness, it is sufficient to upper bound the TCP-friendliness of Robust-AIMD(α, β, ϵ). We hence restrict our attention hereafter to upper bounding the TCP-friendliness of Robust-AIMD(α, β, ϵ).

Consider first, to simplify exposition, the following 2-sender setting. Let p_1 be a TCP sender (i.e., AIMD(1, 0.5)) and p_2 be a Robust-AIMD(α, β, ϵ) sender that competes with p_1 on the link. To analyze the dynamics of the two senders, we consider three possible scenarios for any given point in time t:

(1) Scenario I: No loss event occurs.
(2) Scenario II: Loss occurs, but the loss rate is at most ϵ.
(3) Scenario III: The loss rate is higher than ϵ.

In the first scenario, both senders increase their window sizes (p_1 by MSS and p_2 by α MSS). In the second scenario, the two senders react in an opposite manner: p_2 additively increases its window size by α MSS per RTT, while p_1 halves its window size. In the third scenario, both senders decrease their window sizes (p_1 halves its window size and p_2 decreases its window size by a multiplicative factor of β).

To upper bound the friendliness, we consider the scenario that the link capacity C»αMSS and the buffer size is dominated by the link capacity (i.e., $C >> \tau$). We make the following observations about how dynamics transition between the three scenarios.

- When in scenario I, the two senders will gradually increase their window sizes until, at some later point in time, reaching Scenario II. We denote this by $I \rightarrow II$.
- When in scenario II, the two senders will adjust the sending rates in opposite directions, reaching either Scenario I (if p_1's change to its congestion window dominates p_2's) or Scenario III (otherwise).
- When in Scenario III, both senders will decrease their sending rates, eventually reaching Scenario I.

Thus, any dynamics of interaction between the two senders is expected to result in a sequence of transitions between scenarios of the form $I \rightarrow II \rightarrow I \rightarrow II \rightarrow \ldots \rightarrow I \rightarrow II \rightarrow III \rightarrow I \rightarrow II \rightarrow I \rightarrow II \rightarrow \ldots \rightarrow I \rightarrow II \rightarrow III \rightarrow I \rightarrow \ldots$. An illustration of such dynamics is presented in Fig. 5. We call a subsequence connecting every two appearances of Scenario III a "cycle". To determine the TCP-friendliness of p_2, we compute the average window size of p_1 and p_2 over the course of a cycle and examine the ratio between them. We refer to a transition of the form $I \rightarrow II$ as a sub-cycle.

We observe that in any subsequence of the form $I \rightarrow II \rightarrow \ldots \rightarrow I \rightarrow II \rightarrow I \ldots$, p_2 constantly increases its window size and, moreover, Scenario III will be reached only once the window size of p_1 is below 2αMSS (for only then, halving p_1's window size in Scenario II is dominated by increasing p_2's window size by αMSS. Thus, whenever Scenario I is realized, the sending rates will eventually be so that p_2 completely dominates the link and p_1 is effectively sends at a rate of almost 0 (as $\alpha\text{MSS} \ll C$). Once this happens, p_2 will keep increasing its rate until Scenario III is realized, and so on.

Immediately after reaching Scenario III, the window size of p_2 is $\beta\frac{C+\tau}{1-\epsilon}$ (Lemma 14), while p_1's window size remains effectively 0.

Consider a specific cycle, and let S^i denote the i'th sub-cycle in S. Let x_{L_i} and x_{H_i} denote the smallest window size and the largest window size of p_1 in S^i, respectively, and let y_{L_i} and y_{H_i} denote the smallest window size and the largest window size of p_2 in S^i, respectively. Let T_i denote the duration of sub-cycle S^i. Note that $T_i = x_{H_i} - x_{L_i}$ (as the additive increase for TCP Reno is 1).

Building on the above observations, we now broaden our focus to the more general setting in which m_1 p_1-senders compete with m_2 p_2-senders. Using the same argument as in the proof of Theorem 2, the m_1 p_1-senders can be regarded as a single AIMD($m_1, \frac{1}{2}$)-sender, denoted by P_1, and the m_2 p_2-senders can be regarded as a single Robust-AIMD($m_2\alpha, b, \epsilon$)-sender, denoted by P_2. As in the 2-sender analysis, in every sequence of sub-cycles, P_2 increases its window size until it entirely dominates the link capacity. Hence, the duration of a cycle is $T = \frac{(1-\beta)(C+\tau)}{m_2\alpha(1-\epsilon)}$. Consequently, the average window size of P_1 in one cycle is:

$$\begin{aligned}
&\frac{1}{T}\left(\sum_{x^t=L_1}^{x_{H_1}} x^t + \ldots + \sum_{x^t=L_N}^{x_{H_N}} x^t\right) \\
=&\frac{m_2\alpha}{\frac{C+\tau}{1-\epsilon}(1-\beta)}\sum_{i=1}^{N}\frac{(x_{L_i}+x_{H_i})}{2}(x_{H_i}-x_{L_i}) \\
=&\frac{m_2\alpha}{2\frac{C+\tau}{1-\epsilon}(1-\beta)}\sum_{i=1}^{N}(x_{H_i}^2 - \frac{x_{H_{i-1}}^2}{4}) \qquad (10)
\end{aligned}$$

where the last equality follows from P_1 halving its window size (that is, $x_{L_i} = \frac{x_{H_{i-1}}}{2}$). We next compute x_{L_i} and x_{H_i}. In the beginning of cycle, P_1's window size is (effectively) 0, and so $x_{L_0} = 0$, and P_2 starts at $\beta\frac{C+\tau}{1-\epsilon}$. In the beginning of each sub-cycle S^i, $(C+\tau-(x_{L_{i-1}}+y_{L_{i-1}}))$ is left to consume

before the cycle ends, and the sum of the window increases of the two senders is $m_1 + m_2\alpha$. Thus, each sub-cycle lasts $T_i = \frac{C+\tau-(x_{L_{i-1}}+y_{L_{i-1}})}{m_1+m_2\alpha}$ time steps.

In addition, note that

$$x_{H_i} = x_{L_i} + \frac{C+\tau-(x_{L_i}+y_{L_i})}{m_1+m_2\alpha} \tag{11}$$

Since $x_{L_i} = \frac{x_{H_{i-1}}}{2}$, and as P_2 grows by $m_2\alpha$MSS each RTT, $y_{L_i} = y_{H_{i-1}} + m_2\alpha$, we have that

$$T_i = \frac{C+\tau-(\frac{x_{H_{i-1}}}{2}+y_{H_{i-1}}+m_2\alpha)}{m_1+m_2\alpha} \approx \frac{C+\tau-(\frac{x_{H_{i-1}}}{2}+y_{H_{i-1}})}{m_1+m_2\alpha}$$

where the last step holds true as $\frac{m_2\alpha}{m_2\alpha+m_1} < 1$. To determine x_{H_i}, we write the recurrence relation for x_{H_i} and y_{H_i} to describe how they progress.

$$x_{H_i} = \frac{x_{H_{i-1}}}{2} + m_1\left[\frac{C+\tau-(\frac{x_{H_{i-1}}}{2}+y_{H_{i-1}})}{m_1+m_2\alpha}\right] \tag{12}$$

$$y_{H_i} = y_{H_{i-1}} + m_2\alpha\left[\frac{C+\tau-(\frac{x_{H_{i-1}}}{2}+y_{H_{i-1}})}{m_1+m_2\alpha}\right] \tag{13}$$

After some algebraic manipulation of (12) and (13), we isolate a recurrence relation for x_{H_i} only,

$$x_{H_i} = x_{H_{i-1}}\frac{2m_1+m_2\alpha}{2(m_1+m_2\alpha)} \tag{14}$$

Solving (14) using initial condition $x_{H_1} = \frac{\frac{C+\tau}{1-\epsilon}(1-\beta-\epsilon)m_1}{m_2\alpha+m_1} < \frac{\frac{C+\tau}{1-\epsilon}(1-\beta)m_1}{m_2\alpha+m_1}$ (which is derived from the time it takes to fully utilize the bandwidth) gives an upper bound of x_{H_i} as a function of i,

$$x_{H_i} = \frac{2(1-\epsilon-\beta)\frac{C+\tau}{1-\epsilon}}{2m_1+m_2\alpha}\left(\frac{m_2\alpha+2m_1}{2(m_1+m_2\alpha)}\right)^i \tag{15}$$

$$< \frac{2(1-\beta)\frac{C+\tau}{1-\epsilon}}{2m_1+m_2\alpha}\left(\frac{m_2\alpha+2m_1}{2(m_1+m_2\alpha)}\right)^i \tag{16}$$

Plugging (15) in (10), assigning a total time of $T = \frac{(1-\beta)(C+\tau)}{m_2\alpha(1-\epsilon)}$, and replacing N with ∞ (after N sub-cycles $H_N - L_N$ is effectively zero, as the link is almost utilized and the p_1 sender sends at most 1 MSS), we get that the average window size of P_1 is at most,

$$\begin{aligned}
&\frac{m_2\alpha}{2\frac{C+\tau}{1-\epsilon}(1-\beta)}\left[\left(\frac{\frac{C+\tau}{1-\epsilon}(1-\beta)m_1}{m_1+m_2\alpha}\right)^2 + \right.\\
&\sum_{i=2}^{\infty}\left(\frac{\frac{C+\tau}{1-\epsilon}(1-\beta)2m_1}{2m_1+m_2\alpha}\left(\frac{2m_1+m_2\alpha}{2(m_1+m_2\alpha)}\right)^i\right)^2\\
&\left.-\frac{1}{4}\sum_{i=2}^{\infty}\left(\frac{\frac{C+\tau}{1-\epsilon}(1-\beta)2m_1}{2m_1+m_2\alpha}\left(\frac{2m_1+m_2\alpha}{2(m_1+m_2\alpha)}\right)^{i-1}\right)^2\right]\\
&= \frac{\frac{C+\tau}{1-\epsilon}(1-\beta)3m_1}{2(3m_2\alpha+4m_1)}
\end{aligned} \tag{17}$$

On the other hand, since P_2 starts to increase its window size at $\beta\frac{C+\tau}{1-\epsilon}$ until its window size is $\frac{C+\tau}{1-\epsilon}$ (with a window size increase of $m_2\alpha$ at each step), the average window size of P_2 in a single sub-cycle is,

$$\frac{\frac{C+\tau}{1-\epsilon}(1+\beta)}{2} \tag{18}$$

Since multiple AIMD senders converge to a fair rate configuration amongst themselves [16], and this is also true for Robust-AIMD senders (using the same analysis), to quantify the TCP-friendliness of Robust-AIMD(α, β, ϵ), we need to compute the ratio R between the average window size of any p_1-sender (using (17)) and the average window size of any p_2-sender (18)). This boils down to at most

$$R = \frac{\frac{1}{m_1}\cdot\frac{\frac{C+\tau}{1-\epsilon}(1-\beta)3m_1}{2(3m_2\alpha+4m_1)}}{\frac{1}{m_2}\cdot\frac{\frac{C+\tau}{1-\epsilon}(1+\beta)}{2}} = \frac{3m_2(1-\beta)}{(3m_2\alpha+4m_1)(1+\beta)} \tag{19}$$

In particular, by Equation (19), the ratio for a single Robust-AIMD($1, \frac{1}{2}, \epsilon$) sender and a single TCP Reno sender is $R = \frac{3\cdot(1-\frac{1}{2})}{(3+4)(1+\frac{1}{2})} = \frac{1}{7}$. Note that the ratio R increases with the number of robust-AIMD senders and decreases with the number of TCP senders, so long as the congestion loss is no larger than ϵ (as otherwise all senders decrease their window size). Hence, the worst-case value for TCP-friendliness is achieved when $m_2 = 1$ and m_1 satisfies

$$m_1 + 1\cdot\alpha = \frac{C+\tau}{1-\epsilon} \tag{20}$$

where (20) bounds the number of TCP senders given that each sender's window size is 1 MSS. From (20) it follows that the lowest ratio R is obtained when

$$m_1 = \frac{C+\tau}{1-\epsilon} - \alpha \tag{21}$$

Plugging expression (21) into (19) and rearranging the terms yields

$$R = \frac{3(1-\beta)}{(4(\frac{C+\tau}{1-\epsilon})-\alpha)(1+\beta)}$$

Hence, any ϵ-robust AIMD is at most $\frac{3(1-\beta)}{(4(\frac{C+\tau}{1-\epsilon})-\alpha)(1+\beta)}$-friendly.

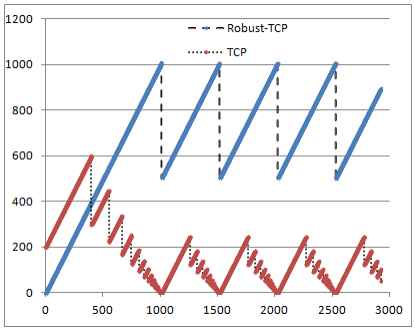

Fig. 5. Dynamics of TCP vs. Robust-AIMD(1,0.5,0.01). The figure plots the windows sizes across time. The link's bandwidth-delay product is 800 MSS and a buffer size is 200 MSS. The Robust-AIMD sender increases its window size until its experienced loss rate is 0.01.

Proof of Theorem 4: When $\epsilon << \beta$, the value of x_{H_i} in the proof of Theorem 3 becomes:

$$x_{H_i} = \frac{2(1-\epsilon-\beta)\frac{C+\tau}{1-\epsilon}}{2m_1+m_2\alpha}\left(\frac{m_2\alpha+2m_1}{2(m_1+m_2\alpha)}\right)^i \approx \frac{2(1-\beta)\frac{C+\tau}{1-\epsilon}}{2m_1+m_2\alpha}\left(\frac{m_2\alpha+2m_1}{2(m_1+m_2\alpha)}\right)^i \tag{22}$$

The proof of Theorem 4 then follows directly from the proof of Theorem 3.

Proof of Theorem 5: Let $\bar{x}_P$ and $\bar{x}_{TCP}$ be the average window sizes of P and TCP, respectively, when competing against each other on the link. Suppose that P is α-TCP-friendly. Then, $\alpha \cdot \bar{x}_P < \bar{x}_{TCP}$. Let $\bar{x}_Q$ and $\bar{x'}_{TCP}$ be the average window size of Q and TCP respectively when they compete each other on the same link. Suppose that Q is more aggressive to TCP. Then $\bar{x'}_{TCP} < \bar{x}_Q$.

Bansal and Balakrishnan [9] showed that given a loss frequency p (the ratio between the number of time steps in which loss occurs and the overall number of time steps), any protocol of type BIN(a, b, l, k) converges to an average window size that is proportional to,

$$\left(\frac{a}{(1-b)p}\right)^{\frac{1}{l+k+1}} \tag{23}$$

for any $l \leq 1, k \geq -1$. Let $M^{a,b,l,k}(p)$ denote $\left(\frac{a}{(1-b)p}\right)^{\frac{1}{l+k+1}}$ where a, b, l, k are the corresponding parameters for protocol M. Since $\alpha\bar{x}_P < \bar{x}_{TCP}$ (independent of p), it follows that for all $p \in (0, 1)$,

$$\alpha \cdot P^{a,b,l,k}(p) < TCP(p) \tag{24}$$

Where TCP(p) is the average window size of TCP Reno for a given p. Similarly,

$$TCP(p) < Q^{a',b',l',k'}(p) \tag{25}$$

Since inequalities 24 and 26 hold for any $p \in (0, 1)$, we have

$$P^{a,b,l,k}(p) < Q^{a',b',l',k'}(p) \tag{26}$$

Therefore, $\alpha \cdot \bar{x}_P < \bar{x}_Q$ for any $p \in (0, 1)$, which implies that P is α-friendly to Q (note that this holds for any number of senders). Since both AIMD and MIMD are special cases of BIN (for $k = 0, l = 1$ and $k = -1, l = 1$ respectively) the theorem follows.

Proof of Theorem 6: Consider a loss-based protocol P that is α-efficient for some $\alpha > 0$, and a latency-decreasing protocol Q. Suppose that a single P-sender is sharing the link with a single Q-sender. As Q is latency-decreasing, there exists some value δ such that when the experienced RTT exceeds $\delta \times 2\Theta$ the Q-sender must decrease its rate. Now, let the size of the buffer τ be such that the when the sum of sending rates is $\alpha(C + \tau)$ the RTT is strictly higher than $\delta \times 2\Theta$. Consider the scenario that the initial congestion window of P is $C + \tau$ and the initial congestion window of Q is 1 MSS (which is assumed to be negligible with respect to $C + \tau$). This initial choice of congestion window sizes captures a scenario in which the P-sender is fully utilizing the buffer when the Q-sender starts sending (or, alternatively, the Q-sender experienced a timeout while the P-sender continues sending at a high rate). We argue that henceforth, the congestion window of the P-sender will always be lower bounded by $\alpha(C + \tau)$ and so the RTT will always be at least $\delta \times 2\Theta$. Consequently, the Q-sender cannot increase its congestion window and must send at its minimum rate. To see why, observe that as P is loss-based, from the P-sender's perspective, this environment is indistinguishable from the environment in which the link capacity is $C + \tau$ and there is no buffer. In addition, so long as the Q-sender sends at a fixed (its minimum) rate, the P-sender cannot distinguish between the (actual) scenario that the link is shared with others and

the scenario that the P-sender is alone on the link (as if there are multiple P senders, their overall rate is fixed and hence the loss rate is not affected). Consequently, α-efficiency dictates that the congestion window of the P-sender remain of size at least $\alpha(C+\tau)$. This, in turn, by our choice of τ, forces the Q-sender to not increase its congestion window size, as otherwise, the RTT will be is strictly higher than $\delta \times 2\Theta$. The theorem follows.

Proof of Theorem 7: Abusing notation, let S^t denote the congestion window of S, the protocol, at time t. We model the environment as an adversary that seeks changes in bandwidth the result in high delays for S. Let A^t denote the bandwidth chosen by the adversary at time t. Since S is α-efficient on average, we are guaranteed that, in the long run, for any sequence of link bandwidths (chosen by the adversary), at least an α-fraction of the link will be utilized on average (see the revisited axiom of efficiency in Section 6.1). Note that if $\alpha B_2 < B_1$, then S can always guarantee the minimum achievable latency (2θ) by constantly choosing $2\theta B_1$. Hence, we will henceforth assume that $\alpha B_2 \geq B_1$. If $2\theta A^t < S^t$ then the experienced latency at time t is $Q^t = 2\theta + \max(\frac{S^t - 2\theta A^t}{2\theta A^t}, 0)$. A's objective is to maximize the average experienced latency. We observe that the optimal decisions of the adversary will always be the either $2\theta B_1$ or $2\theta B_2$ (and no choices of bandwidths in between these two values).

We consider a natural strategy for the adversary, we term the "β-threshold strategy": A chooses some β; if S chooses a congestion window size that is larger than $2\theta\beta B_2$, then A's response is $A^t = B_1$; otherwise, A's response is $A^t = B_2$.

Claim 5 (proven below) states that, at any point in time t, the sender's optimal response to the threshold strategy is either choosing $S^t = 2\theta\beta B_2$, which will not inflict any queuing delay (only propagation delay of 2θ) but provide efficiency of only $\frac{2\theta\beta B_2}{2\theta B_2} = \beta$, or choosing $S^t = 2\theta(\beta+\epsilon)B_2$ (a congestion window that is larger but arbitrarily close to βB_2), which will inflict a latency of $2\theta(1 + \frac{\beta B_2 - B_1}{B_1})$, but provide efficiency of 1.

Let DOWN denote the action $S^t = 2\theta\beta B_2$, and UP denote the action $S^t = 2\theta(\beta+\epsilon)B_2$. As we assume that $\beta < \alpha$ (otherwise the sender can constantly choose DOWN and achieve α efficiency), S has to guarantee average efficiency of α by choosing UP at least once in a while. Since only the proportions between the number of times UP and DOWN are chosen affect the efficiency and the latency (i.e., the order of sender's actions has no effect), we assume, w.l.o.g., time cycles of size t in which S chooses UP only once and DOWN the remainder of the time (i.e., for $t-1$ times steps). Let RTT^{up} denote the RTT when the sender chooses UP, and RTT^{down} denote the RTT when the sender chooses DOWN.

$RTT^{up} > RTT^{down}$ (as there is a queuing delay when the sender chooses UP). We compute the efficiency (see the revised axiom in Section 6.1) as follows

$$\begin{aligned}
&\frac{RTT^{up} \times 1 + (t-1)RTT^{down} \times \beta}{RTT^{up} + (t-1)RTT^{down}} \\
&= \frac{2\theta(1 + \frac{\beta B_2 - B_1}{B_1}) \times 1 + 2\theta(t-1) \times \beta}{2\theta(1 + \frac{\beta B_2 - B_1}{B_1}) + 2\theta(t-1)} \\
&= \frac{\beta B_2 + B_1(t-1)\beta}{\beta B_2 + B_1(t-1)}
\end{aligned}$$

The average efficiency must therefore satisfy,
$\frac{\beta B_2 + B_1(t-1)\beta}{\beta B_2 + B_1(t-1)} \geq \alpha$.

Rearranging the terms, we get

$$t \leq \frac{(1-\alpha)\beta B_2 + B_1(\alpha-\beta)}{B_1(\alpha-\beta)} \tag{27}$$

Since the sender is better off choosing DOWN over UP (so long as the average efficiency requirement is satisfied), for S's optimal strategy, the inequality in (27) is tight, that is,

$$t = \frac{(1-\alpha)\beta B_2 + B_1(\alpha-\beta)}{B_1(\alpha-\beta)} \tag{28}$$

The revised latency-avoiding axiom (Section 6.1) averages over packets. Therefore,

$$\begin{aligned}\bar{Q} &= \frac{Q^{\text{UP}} 2\theta\beta B_2 + Q^{\text{DOWN}}(t-1)2\theta\beta B_2}{2\theta t\beta B_2} \\ &= \frac{2\theta\left(\frac{\beta B_2 - B_1}{B_1} + t\right)}{t} \\ &= \frac{2\theta(1-\beta)\beta B_2}{(\alpha-\beta)B_1 + \beta(1-\alpha)B_2}\end{aligned} \tag{29}$$

where the second equality holds as $Q^{\text{DOWN}} = 2\theta$ and $Q^{\text{UP}} = 2\theta(1 + \frac{\beta B_2 - B_1}{B_1})$. Since A's goal is to maximize latency, its choice of the threshold β maximizes $\bar{Q}$. The maximizer of $\bar{Q}$ is

$$\beta^* = \frac{\alpha B_1 \pm \sqrt{\alpha(1-\alpha)B_1(B_2-B_1)}}{B_1 - (1-\alpha)B_2}$$

where the square root is positive iff $\frac{b}{c} > 1-\alpha$. The corresponding average latency is (by setting β to be β^* in (29) is

$$\bar{Q}(\alpha) = \frac{2\theta B_2\left((1-\alpha)(B_2-B_1)+\alpha B_1 - 2\sqrt{(1-\alpha)aB_1(B_2-B_1)}\right)}{((1-a)B_2-B_1)^2} \tag{30}$$

The lower bound on the average latency bound $Q(\alpha)$ is tight (as the sender employs the optimal strategy), and so the sender cannot experience latency strictly lower than $Q(\alpha)$ (or equivalently, $Q(\gamma)$ for $\gamma < \alpha$, as Q is a monotonic function). According to the revised latency-avoiding axiom (Section 6.1), a γ-latency avoiding protocol satisfies $\bar{Q}(\gamma) = 2\theta(1+\gamma)$. Hence,

$$\gamma = \frac{B_2\left((1-\alpha)(B_2-B_1)+\alpha B_1 - 2\sqrt{(1-\alpha)\alpha B_1(B_2-B_1)}\right)}{((1-\alpha)B_2-B_1)^2} - 1 \tag{31}$$

$$= \frac{B_1(B_2-B_1)+\left(\alpha-\alpha^2\right)B_2^2 - 2B_2\sqrt{(1-\alpha)\alpha B_1(B_2-B_1)}}{((1-\alpha)B_2-B_1)^2} \tag{32}$$

7

Claim 4. *The sender's optimal strategy does not decide DOWN in more than a* $\frac{(1-\alpha)\beta B_2}{(1-\alpha)\beta B_2+(\alpha-\beta)B_1}$ *fraction of the time steps.*

Proof. The fraction of time steps that S chooses DOWN is,

$$\frac{t-1}{t} \tag{33}$$

By (28), the minimal time cycle satisfies $t = \frac{(1-\alpha)\beta B_2 + B_1(\alpha-\beta)}{B_1(\alpha-\beta)}$. Hence, plugging t in (33), the proportion of time steps in which S chooses DOWN is,

$$\frac{t-1}{t} = \frac{(1-\alpha)\beta B_2}{(1-\alpha)\beta B_2+(\alpha-\beta)B_1} \tag{34}$$

□

Claim 5. *Given a β threshold strategy of the adversary, the sender's optimal choice of congestion window is always in $\{\beta B_2 2\theta, (\beta+\epsilon)B_2 2\theta\}$ for an arbitrarily small choice of $\epsilon > 0$.*

Proof. To see this, consider first the scenario in which the sender's congestion window is strictly below $\beta B_2 \times 2\Theta$. Then, as in this scenario the realized bandwidth is B_2, a congestion window of size $\beta B_2 \times 2\Theta$ would have led to the exact same realized link bandwidth, but would have resulted in strictly better link utilization (with the same latency). Now, consider the scenario in which the sender's congestion window is strictly above $(\beta+\epsilon)B_2 \times 2\Theta$. Then, the realized link bandwidth is B_1 and a congestion window of size precisely $(\beta+\epsilon)B_2 \times 2\Theta$ would have led to the exact same realized link bandwidth, but would have resulted in strictly better latency (and the same link utilization, i.e., 100%). □

Proof of Theorem 8: Let

$$F(\alpha) \triangleq \frac{2\theta B_2\left((1-\alpha)(B_2-B_1)+\alpha B_1 - 2\sqrt{(1-\alpha)aB_1(B_2-B_1)}\right)}{((1-a)B_2-B_1)^2} \tag{35}$$

(note that $F(\alpha)$ is the latency function in Theorem 7). Restating Theorem 8, any protocol that is α-efficient suffers latency at least $F(\alpha)$ RTT. We show that for a protocol P that guarantees average latency of Q, the efficiency is no higher than

$$F^{-1}(Q) = \frac{(Q+1)(B_2-B_1)+2B_1-B_2+2\sqrt{B_1(B_2-B_1)Q}}{B_2(Q+1)} \tag{36}$$

Suppose, for point of contradiction, that there exists a protocol P that guarantees average latency Q and efficiency $\alpha > F^{-1}(Q)$. Since $F(\alpha)$ is monotonically increasing in α (for $0 < \alpha < 1$), the sender suffers latency of $F(\alpha) > F((F^{-1}(Q))) = Q$—a contradiction.

Proof of Theorem 9: We use the same model and terminology as in the proof of Theorem 7. The loss rate of the sender is $L(x,B) = 1 - \frac{S^t}{B}$. Since the minimal time cycle for which α-efficiency is guaranteed on average is $t = \frac{(1-\alpha)\beta B_2 + B_1(\alpha-\beta)}{B_1(\alpha-\beta)}$ (the term is derived analogously to equation (28)), the average loss is,

$$\bar{L} = \frac{L^{\text{UP}}\beta B_2 + L^{\text{DOWN}}(t-1)\beta B_2}{t\beta B_2} = \frac{L^{\text{UP}}}{t}$$
$$= \frac{1-\frac{B_1}{\beta B_2}}{\frac{(1-\alpha)\beta B_2 + B_1(\alpha-\beta)}{B_1(\alpha-\beta)}}$$

$L^{\text{UP}} = 1 - \frac{B_1}{\beta B_2}$, $L^{\text{DOWN}} = 0$. The optimal β for the adversary A maximizes $\bar{L}$, which due to the first condition of optimality, satisfies

$$\beta^* = \frac{\alpha B_1(\sqrt{(1-\alpha)B_2(B_2-B_1)} + (1-\alpha)B_2 - B_1)}{(\alpha(1-\alpha))B_2^2 + (1-\alpha)B_1B_2 - B_1^2} \tag{37}$$

(note that $\frac{B_1}{B_2} \le \beta \le \alpha$). The corresponding average loss is thus $\bar{L}^* = \frac{(2-\alpha)B_2 - B_1 - 2\sqrt{(1-\alpha)B_2(B_2-B_1)}}{\alpha B_2}$, and no α-efficient protocol achieves loss rate lower than $\bar{L}^*$.

Proof of Theorem 10: We take the inverse function of $\frac{2\sqrt{L(B_2-B_1)(B_1+B_2L)} - B_1L + B_1 + 2B_2L}{B_2(L+1)^2}$ and apply the same methodology as in the proof of Theorem 8.

A.1 Auxiliary Lemmas

LEMMA 11. *Let A_1 be AIMD-(α_1, β_1) and A_2 be AIMD-(α_2, β_2). Then, in equilibrium, the window size of A_1 is $\frac{(C+\tau)\alpha_1(1-\beta_2)}{\alpha_1(1-\beta_2)+\alpha_2(1-\beta_1)}$.*

The proof is similar to the analysis in [13].

PROOF. Let $\delta(t)$ be the time difference between two consecutive loss events. Let $\Delta(t)$ be the difference in window size between two consecutive loss events. Then,

$$\Delta(t) = \beta_1 x_{A_1}^t + \alpha_1 \delta(t) - x_{A_1}^t \tag{38}$$

where $\delta(t)$ is given by,

$$\begin{aligned} \delta(t) &= \frac{(C+\tau)-(\beta_1 x_{A_1}^t + \beta_2 x_{A_2}^t)}{\alpha_1+\alpha_2} \\ &= \frac{(C+\tau)-(\beta_1 x_{A_1}^t + \beta_2((C+\tau)-x_{A_1}^t))}{\alpha_1+\alpha_2} \end{aligned} \tag{39}$$

Plugging (39) into (38) gives

$$\Delta(t) = \beta_1 x_{A_1}^t + \alpha_1 \frac{(C+\tau)-(\beta_1 x_{A_1}^t + \beta_2((C+\tau)-x_{A_1}^t))}{\alpha_1+\alpha_2} - x_{A_1}^t \tag{40}$$

$$= x_{A_1}^t \left(-\frac{\alpha_1(1-\beta_2)+\alpha_2(1-\beta_1)}{\alpha_1+\alpha_2}\right) + \frac{(C+\tau)\alpha_1(1-\beta_2)}{\alpha_1+\alpha_2} \tag{41}$$

This process converges when $\Delta(t) = 0$. Then,

$$x_{A_1}^* = \frac{(C+\tau)\alpha_1(1-\beta_2)}{\alpha_1(1-\beta_2)+\alpha_2(1-\beta_1)} \tag{42}$$

□

LEMMA 12. *Any loss-based protocol that is α-efficient does not decrease the window size by more than a factor of α, i.e., $x_p^{t+1} \geq \alpha x_p^t$.*

PROOF. Consider a single sender that sends on a link with buffer size $\tau = 0$ and decreases its window size by a factor $\beta < \alpha$ when loss occurs. This would contradict α-efficiency. Now, suppose that there are n senders, each with window size $\frac{C}{n}$, and sender i decreases its window size by a factor $\beta < \alpha$. When the link is over-utilized, the sum of congestion window sizes at the following time step satisfies

$$\sum_{j\in[1,n]} x_j^{t+1} = \sum_{j\neq i} x_j^{t+1} + x_i^{t+1} = \alpha(n-1)x_j^t + \beta x_i^t < \alpha C$$

which is a contradiction to α-efficiency. □

LEMMA 13. *MIMD is 0-TCP-friendly.*

PROOF. We will prove that MIMD is not friendly toward any AIMD protocol. Suppose that an AIMD-(a_1, b_1) sender and an MIMD-(a_2, b_2)-sender compete on a link with no buffer ($\tau = 0$) and the senders are in steady state. Let x_1 and x_2 be the window sizes of the AIMD and MIMD senders respectively upon experiencing loss, that is $x_1 + x_2 = C$. As the senders are in steady state, the AIMD sender satisfies $b_1 x_1 + a_1 T = x_1$, where T is the duration of of time period on which no loss packet occurs. This implies that $x_1 = \frac{a_1 T}{1-b_1}$. Similarly, the MIMD sender satisfies $b_2 x_2 a_2^T = x_2$. However, we now have that $T = log_{a_2}\frac{1}{b_2}$, and it follows that $x_1 = \frac{a_1 log_{a_2}\frac{1}{b_2}}{1-b_1}$ and $x_2 = C - \frac{a_1 log_{a_2}\frac{1}{b_2}}{1-b_1}$. Since x_1 is independent of C, and C can be arbitrarily high, $\lim_{C\to\infty}\frac{x_1}{x_2(C)} = 0$. □

Lemma 14. *The sum of window sizes of any set of β-loss-avoiding P-senders is upper bounded, from some point in time onwards, by*

$$\sum_{i=1}^{n} x_i^t \leq \frac{C+\tau}{(1-\beta)}$$

Proof. The maximal sum of rate is achieved when the loss rate is precisely β. Hence $L(X^t) = 1 - \frac{C+\tau}{\sum_{i=1}^{n} x_i^t} = \beta$ and then $\sum_{i=1}^{n} x_i^t = \frac{C+\tau}{1-\beta}$. □

Lemma 15. *Any loss based protocol that is α-efficient utilizes at least $\alpha(C+\tau)$ of the bandwidth delay product.*

Proof. Consider two distinct links: one has link capacity C and buffer size τ and the other has capacity $C' = C + \tau$ and no buffer ($\tau = 0$). By definition, $L(X^{(t)}, C, \tau) = L(X^{(t)}, C', 0)$, i.e., the loss rate a protocol experiences on both links, as a function of its sending rate, is identical. Consequently, since any α-efficient protocol must utilizes at least $\alpha C' = \alpha(C+\tau)$ of the second link's capacity, any α-efficient protocol must also utilize at least an α-fraction of $(C+\tau)$ for the first link. □

B Protocol Characterization (Table 1)

This section presents the proofs for the results in Table 1.

B.1 link-utilization

- AIMD, CUBIC, and MIMD: The minimal link utilization occurs when all senders decrease their window sizes by a factor b right after a loss event. Then, the sum of window sizes is,

$$\sum_{i\in[1,n]} bx_i = b \sum_{i\in[1,n]} x_i = b(C+\tau)$$

  Hence, these protocols are $\frac{b(C+\tau)}{C} = b(1+\frac{\tau}{C})$-efficient.
- BIN: When a BIN sender encounters a loss, its window size is updated according to $x^{t+1} = x^t - bx^{t\,l-1}$. Rearranging the terms of the update rule we get

$$x^{t+1} = x^t(1 - \frac{b}{x^{t\,1-l}})$$

  Since $x^t > 1$ and $l \leq 1$, $x^{t+1} > x(1-b)$. Therefore, BIN is $(1-b)(1+\frac{\tau}{C})$ efficient.
- Robust-AIMD: Senders decrease their window sizes only if the loss rate is larger than k. Hence at loss rate k, the sum of the senders' window sizes is maximized. This occurs when $k = 1 - \frac{C+\tau}{\sum_{i\in[1,n]} x_i}$. Hence, the maximal sum of window sizes satisfies $\sum_{i\in[1,n]} x_i = \frac{C+\tau}{1-k}$. Then, when loss occurs

$$\sum_{i\in[1,n]} x_i = b\frac{(C+\tau)}{1-k}$$

  Therefore, Robust-AIMD is $\frac{b}{1-k}(1+\frac{\tau}{C})$-efficient.

B.2 loss-avoidance

For each of the non-robust protocols (AIMD, BIN, MIMD, and CUBIC), the maximal potential loss rate is achieved when all senders (of the same protocol) increase their window size from the point where the buffer is full (i.e., $\sum_{i\in[1,n]} x_i = C+\tau$).

- AIMD, BIN, and CUBIC: When the sum of congestion window sizes is increased (additively) by S, the sum of congestion window sizes, at the time step right before packet loss, becomes $C + \tau + S$, and the maximal loss rate is

$$1 - \frac{C + \tau}{C + \tau + S} \tag{43}$$

  For each of AIMD, BIN, CUBIC, we analyze S. Then, plugging S into (43) yields the loss-avoiding factor.
  - Each AIMD-sender increases its window size by a and so $S = na$
  - Each BIN-sender increases its window size by ax_i^k. Hence, the highest sum of congestion window sizes (which occurs when all senders have equal window sizes as $k \leq 1$) is $S = na(\frac{C+\tau}{n})^k$.
  - CUBIC: The duration of the time period between any pair of loss events is $T = (\frac{x_i^{max}(1-b)}{c})^{\frac{1}{3}}$. Therefore, for $T-1$ time steps from the last packet loss no packet loss occurs, and packet loss does occur after T time steps (that is $\sum_{i\in[N]} x_i^T \geq C + \tau$ and $\sum_{i\in[N]} x_i^{T-1} \leq C + \tau$). At time $T - 1$, each sender increases its window size by $x_i^T - x_i^{T-1} = x_i^{max} + c(T - (\frac{x_i^{max}(1-b)}{C})^{\frac{1}{3}})^3 - (x_i^{max} + c((T-1) - (\frac{x_i^{max}(1-b)}{C})^{\frac{1}{3}})^3) = c$ (given $T = (\frac{x_i^{max}b}{c})^{\frac{1}{3}}$). Since all senders increase their window sizes by c, $S = nc$.
- MIMD: Each sender increases its window size by a factor of ax_i. Then, $1 - \frac{C+\tau}{(1+a)(C+\tau)} = \frac{a}{1+a}$
- Robust-AIMD: The maximal loss rate is obtained when all senders increase their window sizes from the point where they endure the maximal loss rate k. The sum of congestion window sizes that induces loss rate of k is $\frac{C+\tau}{1-k}$. If all senders increase their window sizes (additively by a), then the loss rate becomes $1 - \frac{C+\tau}{\frac{C+\tau}{1-k}+na} = \frac{(C+\tau)k+na(1-k)}{(C+\tau)+na(1-k)}$.

B.3 fast-utilization

- AIMD and Robust-AIMD: This follows immediately from the additive increase of both protocols.
- BIN: Consider first $k \geq 0$. Each BIN-sender increases its window size by $\frac{a}{(x_i^t)^k}$. If $k > 0$, then as $x_i^t \geq 1$ (w.l.o.g), the larger x_i^t is, the smaller the increase in windows size, and thus the protocol is 0-fast-utilizing. If $k = 0$, the increase step is constant, and the protocol is a-fast-utilizing.
- CUBIC: The minimal increase in window size is in the "critical point", which is when $T = (\frac{x_i^{max}b}{c})^{\frac{1}{3}}$, and $x_i^T - x_i^{T-1} = x_i^{max} + c(T - (\frac{x_i^{max}b}{C})^{\frac{1}{3}})^3 - (x_i^{max} + c((T-1) - (\frac{x_i^{max}b}{C})^{\frac{1}{3}})^3) = c$.
- MIMD: The increase of the MIMD sender grows exponentially over time and so it is ∞-fast-utilizing.

B.4 TCP-friendliness

- AIMD: Using the same arguments in the proof of Theorem 2, AIMD(a, b) is $\frac{3(1-b)}{a(1+b)}$-TCP friendly.
- MIMD is 0-TCP-friendly (Lemma 13).
- BIN: Both TCP and any Binomial protocol converge to a unique stable state (for $l \in [0, 1]$, $k \geq 0$) [9]. The average window size of TCP as a function of loss frequency p (i.e., the ratio between the number of time steps in which loss occurs and the overall number of time steps) is $\sqrt{\frac{3}{2}\frac{1}{p}}$ (see [36]). By [9], the expected window size of a binomial protocol as a function of loss rate is approximately $E^{\text{BIN}}(x) = \left(\frac{a}{bp}\right)^{\frac{1}{l+k+1}}$. Since both protocols experience the same loss

frequency (that is, the number of packets between two loss events is approximately equal) when they interact, we can derive the TCP-friendliness of the binomial protocol simply by examining the ratio between the average window sizes of the two protocols. This yields $\sqrt{\frac{3}{2}}\left(\frac{b}{a}\right)^{\frac{1}{l+k+1}} p^{(\frac{1}{l+k+1}-\frac{1}{2})}$. Since TCP-friendliness should apply to any $p > 0$, we get that BIN is approximately $\sqrt{\frac{3}{2}}\left(\frac{b}{a}\right)^{\frac{1}{l+k+1}}$-TCP friendly when $l + k \geq 1$, and 0-TCP-friendly otherwise.

- CUBIC: Let t be the number of packets between two consecutive loss events. The average TCP window size is $\bar{x}^{RENO} = \sqrt{\frac{3t}{2}} = 1.22\sqrt{t}$ [24], and the average CUBIC window size is $\bar{x}^{CUBIC} = \sqrt[4]{\frac{c(4-\beta)t^3}{4\beta}}$ [24]. By plugging $\beta = 1 - b$ (as the decrease in [24] is by a factor of $1 - b$) we get $\bar{x}^{CUBIC} = \sqrt[4]{\frac{c(3+b)t^3}{4(1-b)}}$. The ratio between the average window sizes of the two protocols (for any number of senders) is

$$\frac{\bar{x}^{RENO}}{\bar{x}^{CUBIC}} = \frac{1.22\sqrt{t}}{\sqrt[4]{\frac{c(3+b)t^3}{4(1-b)}}} = 1.22\sqrt[4]{\frac{4(1-b)}{c(3+b)t}}$$

  Since the number of packets between two loss events is no more than $C + \tau$, we have that $t \leq C + \tau$. Hence, the t that minimizes the ratio is $t = C + \tau$, which gives

$$\frac{\bar{x}^{RENO}}{\bar{x}^{CUBIC}} = 1.22\sqrt[4]{\frac{4(1-b)}{c(3+b)(C+\tau)}}$$

- Robust-AIMD: Using the same arguments as in the proof of Theorem 3, Robust-AIMD is $\frac{3(1-k)(1-b)}{(4\frac{C+\tau}{1-k}-a)(1+b)}$-TCP-friendly.

B.5 fairness

- AIMD: AIMD converges to a fair state [16]. Similar arguments show that Robust-AIMD is also fair.
- BIN is fair, as showin in [9].
- MIMD: The ratio between the window sizes of any two MIMD-senders remains constant throughout the dynamics, and is determined exclusively by the initial conditions. Since the initial window sizes for two MIMD senders can be arbitrarily far, this implies 0-fairness.
- Cubic is fair, as shown in [24].

B.6 convergence

- AIMD, Robust-AIMD, and CUBIC: These protocols converge to a steady state ([16, 24]). The convergence parameter for these protocols is a function of the decrease step parameter (because the window size before a decrease is maximal and the window size after a decrease is minimal).
- MIMD: The argument is similar to the argument for AIMD, Robust-AIMD, and CUBIC above. The convergence parameter for MIMD is a function of the decrease step parameter (because the window size before a decrease is maximal and the window size after a decrease is minimal).
- BIN decrease the window size by a factor of at most $(1 - b)$ (see arguments regarding link utilization of BIN).

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